Demand Management

- Principles of demand management
- Airline yield management
- Determining the booking limits
  - A simple problem
  - Stochastic gradients for general problems

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Principles of demand management

Issues:

» Customers are not all the same
» The highest paying customers will not fill up available capacity.
» The lowest paying customers will not cover the full costs of operations.
» Marginal costs may be much lower than average costs.
Principles of demand management

■ From microeconomics:

\[
\begin{align*}
\text{Price} & \quad \text{Quantity} \\
\text{P} & \quad \text{Q} \\
\end{align*}
\]

A single price leaves a lot of money on the table. The airlines would like to capture some of this.
We would like to charge a range of prices to capture the diversity of price elasticities.
Modes of control:

» Primal
  • Limit access to the system (now)
  • Reservations (future)

» Dual
  • Pricing
  • Service (waiting in line)

» Informational
  • Advertising
  • Promotions

» Combinations:
  • Coupons - provide discounts (dual) with restricted services (primal) to customers who hold coupons (informational).
Principles of demand management

Market differentiating characteristics used in practice:

» Advance notice (pre-booking, reservations)
» Length of stay, stay over Saturday, etc.
» Willingness to accept nonrefundable tickets

Service
• Additional amenities (first class, business class)
• Pre-board privileges (frequent flier status)
Principles of demand management

Applications of demand management
» Airlines
» Hotels
» Universities (with rolling admissions)
Lecture outline - Demand Management

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Airline yield management

Overview

» Old system:
  • Airlines establish a fare (or two, such as coach and first class)
  • Sell seats at the fare until all seats are sold or the flight departs.
  • Because of no-shows, airlines may sell too many seats to increase load factors.

» Issue:
  • Business travelers are willing to pay much more, but want the flexibility of making plans at the last minute.
  • Different customers want different “products” measured in terms of price and service.

» Solution:
  • Offer blocks of seats at increasingly higher prices. The lower demand for higher priced seats has the effect of making seats available for last-minute business travelers.
Airline yield management

Booking limits

Blocks of seats are offered at different prices:

- $1700
- $1425
- $1150
- $910
- $850
- $775
- $525
Airline yield management

Consider a two-class problem:
Airline yield management

What price to charge?

A high price gets the business travelers, but squeezes out the low-priced segment. But there are not enough business people to fill the plane.

A lower price attracts the personal travelers, but squeezes out the business people.
Different customer classes vary in terms of their booking process.
Airline yield management

Demand management techniques

» Informational
  • Advertising services
  • Discounts

» Dual (pricing)
  • Setting the price for seats
  • Pricing differentiated services

» Primal (booking limits)
  • We have to limit the number of seats we sell at each fare.
Airline yield management

Consider a two-class problem:

Lost customers: | Cheap seats | Expensive seats
---|---|---

How do we decide if we picked the right booking limit?
Airline yield management

- Challenge question: 
  » What happens when the allotment of “cheap seats” is set too high? Too low?
Airline yield management

Notation:

Activity variables:

- $J$ = Set of fare classes. $j = 1$ is the "highest" fare class.
- $D_j(\omega) =$ (random) Demand for fare class $j$
- $f_j(y) =$ p.d.f. of demand for fare class $j$

Decision variables:

- $U_j =$ Limit (upper bound) on number of reservations allowed for fare class $j$.
- $x_j(\omega) =$ Number of reservations made for fare class $j$.

Parameters:

- $C =$ Capacity of the aircraft
- $p_j =$ Fare (price) for fare class $j$ ($p_j \geq p_{j+1}$)
Objective:

We would like to find the booking limits that solve:

\[
\max_{U_1 \ldots U_J} E \left\{ \sum_{j \in J} p_j x_j(\omega) \right\}
\]

subject to limits on how much we can book.

We can think of \( \omega \) as a single flight, and the expectation is a summation over many flights. We cannot maximize the profits for a single flight, but we would like to maximize profits over many flights.
Airline yield management

- Booking limits:
  - Separable booking limits- Booking limit $j$ limits reservations for fare class $j$ alone:
    
    $x_j \leq U_j$
    
    $\sum_{j \in J} U_j = C = \text{aircraft capacity}$

  - Nested booking limits - Booking limit $j$ limits fare class $j$ and all lower fare classes.
    
    $\sum_{m \geq j} x_m \leq U_j$
    
    $U_j \geq U_{j+1}$
    
    $U_1 = C$
A heuristic derivation for booking limits:

Start with a separable (non-nested) policy:

Let:

\[ P_j(x) = \text{Prob}[D_j \leq x] \]

\[ = \int_{y=0}^{x} f_j(y)dy \]

This means that if we set a booking limit at \( U_j \), the probability that we could have exceeded it is \( (1 - P_j(U_j)) \). Now let:

\[ \text{EMSR}_j(U_j) = \text{The expected marginal seat revenue for fare class } j \]

\[ = \text{The value an additional seat allocated for fare class } j \]

\[ = p_j \left( 1 - P_j(U_j) \right) . \]
Airline yield management

We start by expressing the objective function:

\[ F(U) = \max_{U_j} E\{F(U, \omega)\} \]

subject to:

\[ \sum_{j \in J} U_j = C \]

where:

\[ F(U, \omega) = \sum_j p_j \min\{D_j(\omega), U_j\} \]

Now consider the derivative:

\[ \frac{\partial F(U, \omega)}{\partial U_j} = \begin{cases} p_j & D_j(\omega) > U_j \\ 0 & \text{Otherwise} \end{cases} \]

This means that:

\[ \mathbb{E}\left\{ \frac{\partial F(U, \omega)}{\partial U_j} \right\} = p_j \mathbb{P}[D_j(\omega) > U_j] = p_j (1 - P_j(U_j)) \]
We can formulate the problem as an unconstrained problem by relaxing the constraint:

\[ F^L(U, \lambda) = \max_{U_j} E\{F(U, \omega)\} + \lambda \left( C - \sum_{j \in J} U_j \right) \]

At optimality, we expect to find \( \frac{\partial F^L(U, \lambda)}{\partial U_j} = 0 \). This means that:

\[ p_j \left( 1 - P_j(U_j) \right) - \lambda = 0 \]

for each \( j \).

Or:

\[ p_j \left( 1 - P_j(U_j) \right) = \lambda \]

What is the economic interpretation of this equation?
The EMSR algorithm:

For a given value of $\lambda$, find the booking limit $U_j$:

$$P_j(U_j) = 1 - \frac{\lambda}{p_j} = \frac{p_j - \lambda}{p_j}$$

Given $\lambda$, we can find $U_j(\lambda)$ so that this is satisfied. Now we have to pick $\lambda$ so that

$$\sum_j U_j(\lambda) = C$$

Since $\lambda$ is a scalar, just have to try different values.
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Stochastic gradient algorithms

- Nested booking limits:
  » A sample booking process:

<table>
<thead>
<tr>
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Nested booking limits:

Start by defining the cumulative demand:

\[
\hat{D}_j(U, \omega) = \text{Total reservations made of customer classes } j \text{ and } "\text{lower}". \\
= \min \{ D_j + \hat{D}_{j+1}(U, \omega), U_j \}
\]

Note: \( \hat{D}_j(U, \omega) - \hat{D}_{j+1}(U, \omega) \) is the number of fare class \( j \) reservations that have been accepted.

What are we implicitly assuming about the call-in process?
For this problem, we again want to solve:
\[ F(U) = \max_{U_j} E\{F(U, \omega)\} \]

where:
\[ F(U, \omega) = \sum_{j \in J} p_j \left( \hat{D}_j(U, \omega) - \hat{D}_{j+1}(U, \omega) \right) \]

But how do we find the gradient?
Stochastic gradient algorithms

Consider a two class system...

We start with $U_1 = C$.

$$F(U, \omega) = p_1 \left( \hat{D}_1(U, \omega) - \hat{D}_2(U, \omega) \right) + p_2 \left( \hat{D}_2(U, \omega) \right)$$

$$= p_1 \left( \min \{D_1 + \hat{D}_2, U_1\} - \hat{D}_2(U, \omega) \right) + p_2 \hat{D}_2(U, \omega)$$

Let's now assume that with our low "vacationers" fare $p_2$, $D_2 >> U_2$. In this case, we have:

$$F(U, \omega) = p_1 \left( \min \{D_1 + U_2, U_1\} - U_2 \right) + p_2 U_2$$

$$= p_1 \left( \min \{D_1 + U_2 - U_2, U_1 - U_2\} \right) + p_2 U_2$$

$$= p_1 \min \{D_1, U_1 - U_2\} + p_2 U_2$$
Stochastic gradient algorithms

Taking the derivative with respect to $U_2$ gives:

$$\frac{\partial F(U, \omega)}{\partial U_2} = -p_1 I_{\{D_1 \geq (U_1 - U_2)\}} + p_2$$

(Remember: $I_{\{X\}} = 1$ if event $X$ is true, and 0 otherwise. $E(I_{\{X\}})$ is then the probability that $X$ is true.)

Taking expectations gives:

$$E\left[ \frac{\partial F(U, \omega)}{\partial U_2} \right] = -p_1 \text{Prob}[D_1 \geq U_1 - U_2] + p_2$$

Recall that $U_1 = C$. Setting the derivative equal to zero, we get:

$$p_1 \text{Prob}[D_1 \geq C - U_2] = p_2$$
When the problem gets more complicated, we do not get such neat results. Instead, we can resort to our stochastic gradient algorithms.

$$\max E\{F(U, \omega)\} = \max E \left\{ \sum_{j \in J} p_j (\hat{D}_j(U, \omega) - \hat{D}_{j+1}(U, \omega)) \right\}$$

Assume we have an initial vector $U^0$. We can find the best value of $U$ using the stochastic gradient iteration:

$$U^{k+1} = U^k + \alpha^k \nabla F(U^k, \omega^k)$$
Consider the change in a booking limit:

» Assume we change from $U_4=40$ to $U_4=41$.

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Change in profits is $p_4-p_3$.

What if $U_3$ was not binding?
Stochastic gradient algorithms

Consider the change in a booking limit:

» What if $U_3$ is not binding, but $U_2$ is?

Now the change in profits is $p_4 - p_2$.

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Stochastic gradient algorithms

Consider the change in a booking limit:

» What if nothing else is binding?

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Now the change in profits is $p_4$. 
Consider the change in a booking limit:

What if $U_4$ is not binding, but $U_2$ is?

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Now there is no change in profits.
Stochastic gradient algorithms

So we have a formula for the gradient when increasing $U_i$:

Let:

$$X^+(\omega) = \begin{cases} p_i & \text{If booking limit } U_i \text{ is binding.} \\ 0 & \text{Otherwise.} \end{cases}$$

$$X^-(\omega) = \begin{cases} p_j & \text{If } U_i \text{ is binding, and } U_j \text{ is binding, and } j < i \text{ is the first class} \\ 0 & \text{If no other booking limit for a higher fare class is binding.} \end{cases}$$

We can now express the gradient using:

$$\frac{\partial F(U, \omega)}{\partial U_i} = X^+(\omega) - X^-(\omega).$$

Since $X^+$ and $X^-$ depend on the sample realization of demands, they are random variables.
Higher demand for fare class 1

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Lecture outline - Demand Management

- Issues in demand management
Extensions

■ The no-show problem
  » How can we manage this?

■ The fairness issue
  » Paying different amounts for the “same” seat
  » In what way are seats different?

■ Managing complexity
  » 1000 markets
  » 30,000 fares!