ADAPT: A Price-stabilizing Compliance Policy for Renewable Energy Certificates: The Case of SREC Markets

Javad Khazaei and Michael Coulon and Warren B. Powell

February 15, 2016

Abstract

Currently most Renewable Energy Certificate (REC) markets are defined based on targets which create an artificial step demand function resembling a cliff. This target policy produces volatile prices which make investing in renewables a risky investment. In this paper, we propose an alternative policy called Adjustable Dynamic Assignment of Penalties and Targets (ADAPT) which uses a sloped compliance penalty and a self-regulating requirement schedule, both designed to stabilize REC prices, helping to alleviate a common weakness of such markets. To capture market behavior, we model the market as a stochastic dynamic programming problem to understand how the market might balance the decision to use a REC now versus holding it for future periods (in the face of uncertain new supply). Then, we present and prove some of the properties of this market, and finally we show that this mechanism reduces the volatility of REC prices which should stabilize the market and encourage long-term investment in renewables.
Keywords: Compliance Policy, Renewable Energy Certificates, Stochastic Dynamic Programming, Price Volatility, Mechanism Design

1 Introduction

To promote the use of renewable sources of energy such as wind, solar, geothermal and biomass, Renewable Portfolio Standards (RPSs) have been implemented in a number of states in the US and other countries. These are sets of regulations requiring increased generation from renewable energy sources usually by obligating Load Serving Entities (LSEs) to obtain a percentage of their generation from renewable sources. A Renewable Energy Certificate (REC) market is one tool used by many states to implement these policies. According to a set of regulations, certified renewable generators are credited with RECs for each unit of electricity generation. All LSEs are required to comply with their regulatory obligations by submitting enough RECs each Energy Year (EY), or otherwise they are charged with a penalty called the Alternative Compliance Payment (ACP) for each REC they are short. These market-based tools are expected to provide a more efficient, competitive and innovative environment for increasing renewable energy supply and decreasing the cost of generation in comparison with other regulatory tools such as feed-in tariffs (Burns and Kang (2012); Naroditsky (2013)).

This particular market design, however, as we discuss more extensively in the next section, can result in undesirably volatile prices. Although REC markets incentivize investing in renewables, excess volatility can affect the amount of investment negatively, and is thus a frequent concern of policymakers. Indeed, authors who advocate fixed-price environmental policy tools like carbon taxes often cite this weakness of market-based mechanisms (see e.g. Jacoby and Ellerman (2004)), while the European Union's Emissions Trading Scheme (ETS) faced heavy criticism in its early years as prices fell rapidly towards zero. Price volatility in both REC and carbon markets is mostly attributed to the artificial vertical demand curve imposed by regulations. A number of papers (such as Felder and Loxley (2012); Berry (2002); Kildegaard (2008); Marchenko (2008)) discuss extensively the problems arising from a vertical demand curve (or a cliff policy) such as an uncompetitive market, volatile prices, higher cost of investment (due to higher risk), and difficult policy evaluation. Various investigations have also been made into ways of reducing price volatility, such as through banking (Maeda (2004)), or by trading in financial options if available (Xu *et al.* (2014)). We aim to add to this literature with a new design for REC markets, accompanied by a rigorous mathematical analysis of its implications.

In this paper, we propose an Adjustable Dynamic Assignment of Penalties and Targets (ADAPT) policy by introducing both a sloped penalty function and an adaptive mechanism for requirements which can adjust to supply and demand imbalances. Under this sloped penalty proposal, the effective ACP is a function of the total submitted RECs in each energy year. While the requirement is not directly a function of the submitted or generated RECs, it can also be chosen to be a function of last year's surplus (or shortage). The slope of the penalty function and the sensitivity to last year's surplus are both tunable regulatory parameters. We show in this paper that this mechanism can be used by policy makers to dampen the volatility of market prices which should encourage long-term investments in solar. We conduct our study of the ADAPT policy using the same generation model as Coulon *et al.* (2015). However, by incorporating a sloped penalty function along with banking between years, optimal compliance decisions each period are non-trivial, requiring substantially more analysis. Moreover, instead of modeling each vintage year separately as in Coulon *et al.* (2015), we adapt the dynamic program to solve for prices simultaneously across all vintage years, over a long time horizon.

This paper makes the following key contributions.

1. We propose an innovative and flexible regulatory policy (that we call ADAPT), which can be used to encourage markets to achieve specific goals (e.g. energy from renewables, limiting emissions, ethanol targets, recyclable garbage, or water usage), without the instability often witnessed in classical "cliff" designs for pre-determined quantity targets. The ADAPT policy is easily integrated into current markets, and represents a generalizable concept for hitting quantitative targets. Through a set of empirical and theoretical results, we show that the ADAPT policy produces prices that are dramatically more stable than would occur assuming optimal behavior with a cliff policy.

2. We derive an optimal policy for submitting RECs to the market (while banking others), which captures the collective market behavior. We describe a series of structural results to accelerate the calculations, and then prove several properties of the optimal policy. These include the following:

(a) We demonstrate how the optimal submission under ADAPT is chosen by market participants to balance prices through time.

(b) We show that the prices of RECs of different vintages (RECs generated in different years) are the same under typical market conditions.

(c) We prove a property of the optimal solution that reduces the decision variable's dimensionality across vintages to a scalar. Dimensionality of the state variable is also shown to reduce to a scalar under certain typical conditions.

(d) We prove that the total penalty payments under the sloped policy is bounded from below by the total payments of the cliff policy for a given submission level.

3. We conduct extensive numerical experiments and simulations which confirm and further illustrate the important properties of the model derived in previous sections.

While we focus on REC markets here (and more specifically SRECs), techniques developed here are arguably transferable to other environmental markets and even to other commodity markets, assuming appropriate modifications or extensions. Firstly, cap-and-trade markets for carbon emissions are a natural link, due to their related market designs and compliance features. An ADAPT-like framework could certainly be envisioned for such markets, and indeed the regulators of carbon markets have recently experimented with various tools to stabilize markets, including price floor mechanisms (e.g. California, the UK) and the new EU ETS Market Stability Reserve (starting 2018), designed to offset long-term imbalances via a self-regulating system, avoiding the need for intervention. Similarly, the Massachusetts SREC market sets requirements each year that respond to the surplus or shortage of the previous year, using a more complicated formula than in our own analysis, includng links with of SREC auction results. To our knowledge, the idea of a sloped alternative to a vertical demand curve has not been trialed in REC or carbon markets, but has been implemented in the capacity markets of NYISO and New England ISO (see Cramton and Stoft (2005); Hobbs *et al.* (2007); Stoddard and Adamson (2009) for discussion). Such examples illustrate the willingness of policymakers to investigate and at times implement innovative new market designs.

While only very limited literature on stochastic modeling of SREC markets (see Amundsen *et al.* (2006); Coulon *et al.* (2015); Hustveit *et al.* (2015)), carbon emissions markets have attracted much more attention (see Carmona *et al.* (2010); Hitzemann and Uhrig-Homburg (2014)), including recent studies of the Market Stability Reserve's likely impact on prices Holt and Shobe (2016); Kollenberg and Taschini (2015)). However, various key differences between carbon and REC markets require careful consideration, including the opposite roles for supply and demand, the typical 'withdrawal' rule reducing incentives for banking, the possibility of unlimited banking, and the wide range of factors affecting carbon emissions abatement, notably volatile fuel prices and power demand. We argue that SREC markets provide an excellent case study for such modeling techniques, with simpler underlying factors (e.g., solar generation), relative separation from other energy markets (at least for current low levels of renewables penetration) and more transparent data facilitating empirical studies and model calibration, as performed in Coulon *et al.* (2015) and Hustveit *et al.* (2015).

Table 1: The current requirement and penalty levels (last changed in 2012)

Energy Year	2012	2013	2014	2015	2016	2017	2018	2019	2020
Target (% supply)			2.05%	2.45%	2.75%	3.00%	3.20%	3.29%	3.38%
Target (1000s MWh)	442	596	1,708	2,072	2,360	2,614	2,830	2,953	3,079
SACP	\$658	\$641	\$339	\$331	\$323	\$315	\$308	\$300	\$293

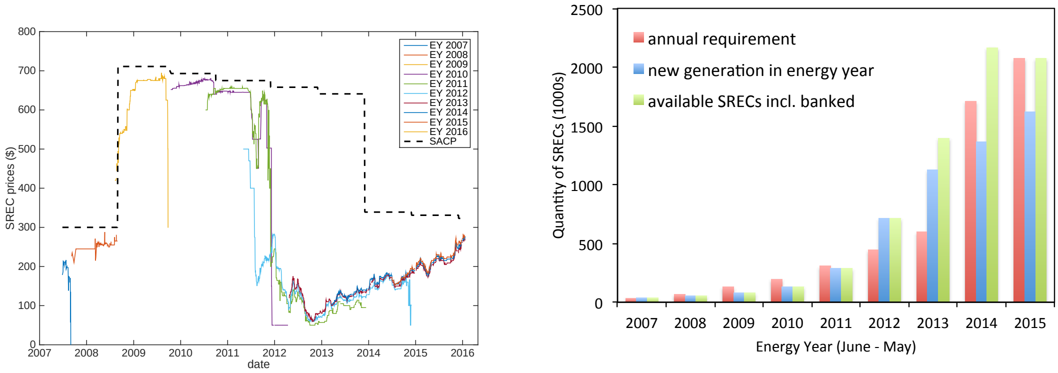
Moving slightly further afield, a large literature on the 'theory of storage' exists for storable physical commodities, such as agricultural, metals or energy commodities (see e.g. Scheinkman and Schechtman (1983); Deaton and Laroque (1996)). Such approaches, called 'structural models' by Pirrong (2014), are based on the idea of a storage decision at each time period (often by a representative agent or social planner), and solved via dynamic programming techniques. In this way, our approach under the APADT framework here is closely linked to this class of models, with the commodity's consumption and storage decisions analagous to our SREC submission and banking decisions, albeit with different seasonal patterns and frequencies. Different harvests of perishable commodities are similar to different SREC vintages, and the same fundamental intertemporal tradeoffs apply, namely that storing more means consuming less now, producing higher prices now (higher cost of satisfying consumption or compliance), but lower prices in the future. However, for agricultural commodities, such models rely crucially on specifying an unobservable inverse demand curve, again giving the SREC case study a significant advantage since the SACP function is set artificially by a regulator.

This paper is organized as follows. In Section 2, we introduce and discuss the market design, regulations, and performance of the New Jersey (NJ) SREC market, including the problems resulting from the current market design. In Section 3, we introduce the ADAPT policy and describe how it can be implemented. In Section 4, we formulate the collective behaviour of market participants using a stochastic dynamic programming model. In Section 5, we characterize and prove some of the properties of the REC markets. In Section 6, we detail our methodology for solving this stochastic dynamic program. In Section 7, we use parameters estimated from New Jersey to perform experiments on different aspects of the ADAPT policy in comparison to the current cliff mechanisms. In Section 8, we provide our concluding remarks.

2 Case Application: The New Jersey SREC Market

In the US, nearly 30 states have established RPSs and some of these use multipliers (e.g. 2 or 3 credits per MWh) to specifically promote investment in solar energy. Nonetheless, many LSEs prefer to use less expensive and more productive sources of renewable energy such as wind energy to meet their REC obligations. Therefore, to even further promote the use of solar energy, 14 states have established separate solar set-asides and tradeable SRECs, and have been more successful in increasing investment in solar generation (Wiser *et al.* (2011); Witmer *et al.* (2012); Bird and Reger (2013); Bird *et al.* (2011); Glickstein (2013)).

The New Jersey (NJ) SREC market is the biggest in the US, has recorded prices near \$700 per SREC, and has the most ambitious target of over 4% of electricity from solar by 2028. SREC generation has grown very rapidly from around 30,000 SRECs in EY2007 to more than



(a) Daily average prices

(b) Monthly SREC issuance rate

Figure 1: Historical NJ SREC prices (left) and annual generation compared to requirement levels (right)

1,000,000 in EY2013. Each energy year represents the twelve month period ending on May 31st of the named year. Table 1 shows the requirement (currently set in terms of percentage of overall supply, with projected absolute numbers) and Solar ACP (SACP) levels according to the latest rule change in 2012 (Bird and Reger (2013); DESIRE (2013); NJCEP (2013)).

The rules of the current SREC market in New Jersey can be summarized as follows:

- For each MWh of solar electricity generated, one SREC is issued to the owner of the plant.
- For several years into the future, the government sets targets for consumption of electricity from solar. These requirements are currently represented as a percentage of the total load.
- All LSEs, primarily utilities, must meet their requirement by submitting sufficient SRECs each year. Otherwise, they need to pay a fixed penalty (or SACP) for each MWh they fall below the target. They are free to generate SRECs themselves or to buy from other SREC generators.
- Finally, SRECs can be banked and used for a few years in the future. In the current NJ SREC market, SRECs can be used for four years in addition to their production year.

Figure 1a shows historical SREC prices in NJ between 2007 and 2016, while Figure 1b shows generation levels, and total banked supply relative to annual targets. Shortly after its introduction, the NJ SREC market sustained very high prices close to its SACP level, as the market failed to meet the targets. More recently, the market was characterised by oversupply in 2012-13, leading to low prices, a rule change to increase the requirement, and less new solar generation, before finally swinging back towards undersupply very recently. Several rapid price drops and jumps can be observed in the historical prices. Ohio and Pennsylvania SREC markets have both also witnessed price drops from above \$300 to under \$50 within 18 month periods. A market with such volatile prices is a risky environment for investors and thus decreases competition and increases the cost of generation (Felder and Loxley (2012)).

To reduce the volatility of SREC prices and thereby reduce the risk of solar investments, several rule changes have been introduced in the NJ SREC market. For example, higher SACP

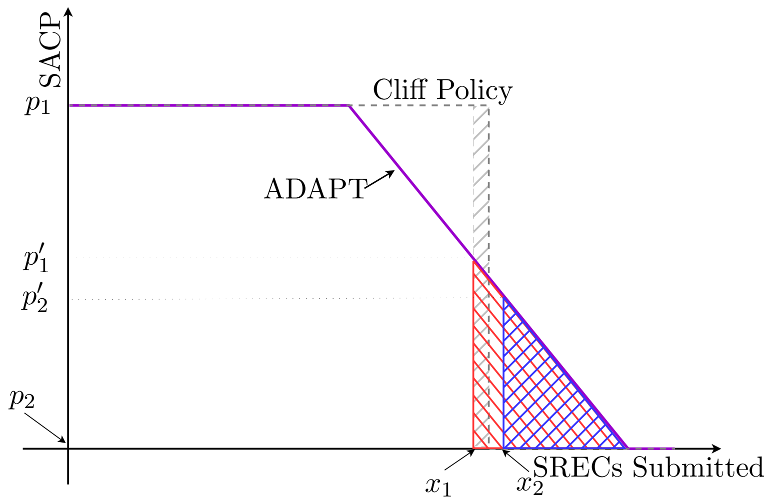


Figure 2: Price formation for two submission scenarios x_1 and x_2 under ADAPT and the cliff policy. The hashed areas represent total penalty payment in each case.

levels and the possibility of banking (for two years) were introduced in 2008. In 2012, the banking horizon was extended to five years, along with increases in the targets. The frequency of changes in the market mechanism shows that these policy adjustments have not been a long-term solution and that market design has some room for improvement. To alleviate these problems, various alternatives are discussed by Felder and Loxley (2012), including price floors, long-term contracts and increased banking years. In addition they briefly suggest the use of a downward sloping demand curve to set non-zero SACP levels even when generation far exceeds the annual SREC requirement. However, as their exponential curve never reaches zero, there is no way to achieve compliance in such a scheme, removing somewhat the notion of a true quantity target. Moreover, they provide only a rough qualitative sketch of the idea, leaving out the details of the implementation, and making no attempt to model resulting price dynamics.

3 ADAPT: A Price-stabilizing Compliance Policy

In this section, we introduce a new class of regulatory policies for computing alternative compliance penalties that allows regulators to avoid excess price volatility, which can discourage investment, and keep it within some tolerance levels. We provide a description of the market design, which we refer to as ADAPT, followed by a rigorous mathematical model in Section 4 which is used to understand the behaviour and price dynamics induced by such a design. We present the ADAPT policy in the framework of SREC markets, but its application could be extended to other environmental markets, such as cap-and-trade markets for emissions.

The standard policy for enforcing compliance used in New Jersey and elsewhere looks like a cliff (see Figure 2), where the SACP is assessed if usage is less than the target, dropping to zero if usage meets or exceeds the target. The first distinguishing feature of ADAPT is to replace the cliff with a downward sloping function, as illustrated in Figure 2. The idea is to reduce the large uncertainty in penalty payments that naturally arises with a cliff-style target policy.

Figure 2 shows a step SACP function i.e. the current mechanism (the solid line), and its counterpart, a sloped SACP function (the purple line). In this figure, x_1 and x_2 represent two scenarios of SREC submissions, with x_1 slightly below the requirement and x_2 slightly above the requirement. Also, p_1 and p_2 are the penalty values of the current mechanism for x_1 and x_2 respectively, with p_1 an extremely high value and p_2 at zero. This means small changes in generation can result in high price volatility. On the other hand, with the sloped SACP function, the same submissions x_1 and x_2 result in penalties p'_1 and p'_2 that are much closer together, providing some initial intuition as to why ADAPT produces more stable prices.

Also, the hashed gray area represents total penalty payment in the current mechanism for x_1, while total penalty payment for x_2 is zero. Note that the SREC requirement under the sloped mechanism (i.e. the right end of the sloped section) is more than the requirement of the current mechanism and so the penalty is paid for a higher number of SRECs (but a smaller penalty value for each unit). The hashed red and blue areas show the total penalty payment under the sloped mechanism for x_1 and x_2 respectively. The intuition behind this penalty payment (v.s. a rectangular payment in which the same SACP price p' is paid for all SRECs short of the requirement) is that in the slope region each SREC further from the right end of the sloped area should be penalized gradually more and more until ADAPT finally starts to penalize each one at the full SACP. This construction allows us to interpret the SACP function as a marginal demand curve, for which a total penalty equal to the area under the curve must be paid.

The sudden price drop from a level close to the SACP to a level close to zero in the cliff policy happens specifically when transiting to a new year. For example, while the probability of the current level of generation not meeting this year's requirement may be high (i.e. SREC prices are high for this year), this probability may be low for the next year. This behavior can occur because targets are different every year, and generation may overtake a future target, producing much lower prices. The sloped SACP function can mitigate this problem by avoiding the binary nature of the cliff policy. Nonetheless, long-term imbalances between supply and demand may still develop, and therefore an additional tunable policy feature may also be incorporated to automatically redefine next year's requirement level, as we shall explain below.

The rules of the proposed ADAPT market can be summarized as follows.

- As usual, one SREC is issued per MWh to solar power generatators.
- The regulator sets SREC requirement levels for several years in the future. However, the requirement levels can be automatically adapted to generation levels according to the market performance in the last year. For example, in case of a surplus (shortage) in the last year, requirements may be increased (decreased) according to a formula.
- All Load Serving Entities (LSEs) must meet their requirement by submitting sufficient SRECs each year, or else pay a penalty for each MWh they fall short. Penalties, however, are calculated based on a sloped SACP function as in Figure 2; the more SRECs submitted, the lower the penalty value per SREC (assuming being in the sloped region of the function).
- A generator or LSE can bank SRECs to the following year, effectively increasing their penalty now and thus raising the value of the SRECs being submitting. This allows

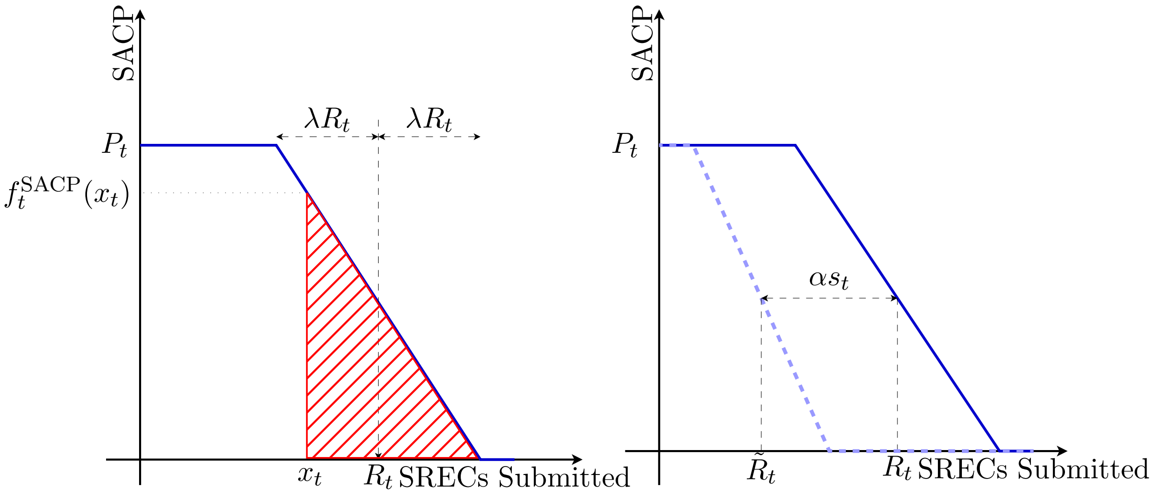


Figure 3: Sloped SACP function (left): the hashed area shows total market penalty payment and x_t represents the total submitted SRECs at the compliance time t ($t \in \mathbb{N}$). Adaptive requirements (right): how R_t is calculated from the fixed base requirement $\tilde{R}_t$ and surplus from the last compliance time (s_t).

SREC holders to more precisely balance the value of SRECs in the future against the price they receive now, but only within a specified range (due to the limited lifetime of each SREC).

Example 3.1 *Assume that the sloped SACP function is defined as represented in Figure 3 (the left plot) with $\lambda = 0.1$. Let $t = 14$ and 15 represent the compliance times of EY2014 and EY2015 respectively. According to the values given in Table 1, the effective requirements of the sloped mechanism (the right end of the sloped area) for EY2014 will be equal to*

$$(1 + \lambda)R_{14} \quad = \quad 1,878,724.$$

Also the maximum (but not the effective) penalty P_{14} and P_{15} is equal to the current SACP levels \$339 and \$331 respectively. Now assume that a total of 2,000,000 SRECs are available before the compliance time of EY2014. We call this value the number of banked SRECs at time $t = 14$, and represent it by $b_{14} = 2,000,000$. Assume that all market participants submit a collective of x_t = 1,800,000 SRECs to the regulators, and the remaining SRECs are banked to be used in the future. According to the sloped SACP function, for this aggregate amount of submission, the SACP is equal to \$78. This means market participants are obliged to pay a total amount of $0.5 \times 78 \times 78,724$ dollars. SRECs are banked because their expected future price is higher than or equal to the penalty of \$78. (Note that in practice SACPs paid would be linked to individual agents' decisions and indeed each LSE faces its own individual requirement, but the existence of the market to allow trading of SRECs up until compliance time should ensure that all parties ultimately choose to pay the same penalty.)

Here, total available SRECs at $t = 14$ exceeds the target $R_{14} = 1,707,931$. Let s_t represent this surplus (or shortage if negative) from the last compliance time. In this case,

$$s_t = b_{14} - R_{14} = 292,069, \qquad t \in (14, 15].$$

In an adaptive requirement scheme, we increase the next year's requirement by a portion of s_t *(as shown in the right plot of Figure 3). For example, for* $\alpha = 0.5$*, we obtain:*

$$R_{15} = \tilde{R}_{15} + \alpha s_{15} = 2,071,803 + 0.5 \times 292,069 = 2,217,838,$$

where $\tilde{R}_{15}$ *is equal to the already announced requirement for EY2015 (from table 1). Note that in the previous period we used* $R_{14} = \tilde{R}_{14}$*, effectively assuming that* $s_{14} = 0$.

We now develop a mathematical model of this mechanism to investigate the effects of the sloped SACP function and adaptive requirements as well as other policy variations (and their combinations), such as number of banking years. Readers should be aware that the current NJ market design is a special case of this general model, in which the slope factor and requirement adaptation factor are set to zero and SRECs are allowed to be banked for four years.

4 Mathematical Model

We consider a market with a sloped SACP function, adaptive requirements, and the possibility of banking for several years. We use the following notation.

Indices (t, y)**:** We index time by t, and energy year by y. In our implementation of the model, the smallest time step is assumed to be a period of one month (i.e. $\Delta t = \frac{1}{12}$), matching the observation frequency of NJ generation data. Each energy year $y \in \mathbb{N}$ is associated with the time interval $(y-1, y]$, while $t = y$ determines the compliance time of energy year y. For example, time $t = 6$ corresponds to the end of energy year 2006 (May 31 2006). We also use y to index the vintage year of SRECs.

Prices $(p_{t,y})_y$**:** The market price at time t of an SREC of vintage year y, which is endogenously determined via our dynamic programming approach, and is a function of the state variable.

Parameters $(T, \tau, \lambda, \alpha, \tilde{R}_y, P_y, r)$**:**

- T: The planning horizon or scheduled length of market existence (could also be ∞).
- τ : The maximum number of years (compliance times) that an SREC can be banked.
- λ $(0 \leq \lambda \leq 1)$: The parameter determining the shape and slope of the SACP function (see figure 3). If $\lambda = 0$, we obtain the current cliff policy.
- α $(0 \leq \alpha \leq 1)$: The parameter representing the portion of last year's surplus (shortage) to be added to (deducted from) the base requirement.
- $\tilde{R}_y$: Number of SRECs required for energy year y, determined by the regulation and announced in advance. This, however, is not the effective requirement R_y, which is determined and updated according to market performance.

- P_y : The penalty (or maximum penalty under ADAPT) set by the regulation to be paid for each SREC short of the requirement R_y in energy year y. We assume that P_y is non-increasing in time, as is typical, and has been the case (and often decreasing) throughout all regulation changes in the NJ SREC market. (Note that the jump upwards in 2008 in NJ was instead a sudden rule change and shift in the entire penalty schedule.)

- r: Interest rate.

State variables: The state variable $S_t = ((b_{t,y})_y, \hat{g}_t, s_t)$ is defined as follows.

- $(b_{t,y})_y$: Total number of accumulated or banked SRECs from different vintages at time t. Also, let $\bar{b}_t$ denote the total number of accumulated SRECs (valid for trading) at time t. If SRECs can be banked for τ years, at time t we have

$$\bar{b}_t = \sum_{y=\max\{1,\lceil t\rceil-\tau\}}^{\lceil t\rceil} b_{t,y}.$$

- $\hat{g}_t$: The installed capacity of SREC generation at time t.

- s_t : The surplus (or shortage) of SRECs from the last compliance time known at time t ($s_t = \bar{b}_{\lceil t\rceil-1} - \tilde{R}_{\lceil t\rceil-1}$). This variable is needed for determining adaptive requirements (for $\alpha > 0$). Note that s_t cannot always be obtained from $(b_{t,y})_y$, e.g., in the case of a shortage ($s_t \leq 0$).

Decision variables $((x_{t,y})_y)$:

- $(x_{t,y})_y$: The number of SRECs to be submitted to the market at time t from different vintage years. We represent the total number of SRECs submitted at time t by $\bar{x}_t$ defined as

$$\bar{x}_t = \sum_{y=\max\{1,\lceil t\rceil-\tau\}}^{\lceil t\rceil} x_{t,y}.$$

Decisions will be made by a policy π using a function $X_t^\pi(S_t)$ to be determined later.

Exogenous information processes (W_t): The exogenous information process $W_t = \varepsilon_t$ is defined as the random variable indicative of noise in generation. Let $\omega \in \Omega$ be a sample path for $(W_1, \ldots, W_T)$. Let $\mathcal{F}_t = \sigma(W_1, \ldots, W_T)$ be the sigma-algebra on Ω, and let $\mathcal{P}$ be the probability measure on $(\Omega, \mathcal{F})$, giving us a probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Throughout our presentation, we assume that any variable indexed by t is $\mathcal{F}_t$-measurable, and that all risk premiums are zero (i.e., $\mathcal{P}$ is the risk-neutral measure).

Other functions $(R_t(s_t), f_t^{\mathbf{SACP}}(\bar{x}_t))$:

- $R_t(s_t)$: The number of SRECs required by the regulation for time period t, defined as

$$R_t(s_t) = \begin{cases} \tilde{R}_t + \alpha s_t & t \in \mathbb{N}, \\ 0 & t \notin \mathbb{N}. \end{cases}$$

- $f_t^{\text{SACP}}(x_t)$: The SACP function, determining the penalty price for any value of submission x_t at time t. This is the (artificial) inverse demand function of SRECs and at any compliance time ($t \in \mathbb{N}$) it can be represented by ($\lambda > 0$):

$$f_t^{\text{SACP}}(\bar{x}_t) = \begin{cases} P_t, & \bar{x}_t < (1-\lambda)R_t, \\ P_t - \frac{P_t}{2\lambda R_t}(\bar{x}_t - (1-\lambda)R_t), & (1-\lambda)R_t \leq \bar{x}_t < (1+\lambda)R_t, \\ 0 & (1+\lambda)R_t \leq \bar{x}_t. \end{cases}$$

  For other time periods $t \notin \mathbb{N}$, we define $f_t^{\text{SACP}}(\bar{x}_t) = 0$. This definition provides a cleaner, more general model without the need to discriminate between compliance and non-compliance times (thus also linking to other markets where storage decisions may be made each period).

Transition function ($\mathbf{S}^M$): We represent this generically using $S_{t+\Delta t} = S^M(S_t, (x_{t,y})_y, W_{t+\Delta t})$. Variables $\hat{g}_{t+\Delta t}$ and $b_{t+\Delta t}$ both depend on the SREC generation model, for which we follow the approach of Coulon *et al.* (2015), supported by empirical evidence from New Jersey. The model consists of seasonality, noise, and a price-dependent expected generation growth rate to capture feedback onto new supply from current SREC prices. (More generally, Coulon *et al.* (2015) allow for dependence on lagged SREC prices or historical average prices to reflect solar construction time, but for simplicity and dimension reduction we use only current prices.) The instantaneous generation rate g_t is given by

$$g_t(p, \varepsilon_t) = \hat{g}_t \exp\Big(a_1 \sin(4\pi t) + a_2 \cos(4\pi t) + a_3 \sin(2\pi t) + a_4 \cos(2\pi t) + \varepsilon_t\Big). \tag{1}$$

while the related installed generation $\hat{g}_t$ evolves according to

$$\hat{g}_{t+\Delta t} = \hat{g}_t \exp(a_5 \Delta t + a_6 p_t \Delta t). \tag{2}$$

The number of banked SRECs $b_{t+\Delta t,y}$ can be obtained from

$$b_{t+\Delta t,y} = \begin{cases} 0 & y < \lceil t+\Delta t \rceil - \tau, \\ b_{t,y} - x_{t,y} & \lceil t+\Delta t \rceil - \tau \leq y < \lceil t+\Delta t \rceil, \\ b_{t,y} + g_t \Delta t - x_{t,y} & y = \lceil t+\Delta t \rceil, \\ 0 & y > \lceil t+\Delta t \rceil. \end{cases}$$

The four cases above correspond respectively to (i) vintages fully expired; (ii) older vintages still trading; (iii) the most recent vintage; and (iv) vintages yet to exist. Between compliance dates $b_{t,y}$ only changes for the current vintage as new SRECs are generated, while at compliance dates total submissions for all existing vintages must be accounted for. Based on the definition of the SACP function, no SRECs are submitted at non-compliance times ($x_{t,y} = 0$ if $t \notin \mathbb{N}$).

Finally, surplus can be updated from

$$s_{t+\Delta t} = \begin{cases} \bar{b}_t - \tilde{R}_t & t \in \mathbb{N}, \\ s_t & t \notin \mathbb{N}. \end{cases}$$

This is only a surplus (or shortage) relative to the base requirement schedule $\tilde{R}_t$, set in advance. Hence it is possible that $s_t > 0$ but the market is short of the adjusted target. ie. $\tilde{R}_t < \bar{b}_t < R_t$.

Objective function: At each time step, the collective behaviour of a competitive market maximizes social welfare or equivalently here, minimizes total cost of compliance. Since the definition of social welfare relies on integrating under an artificial demand function, we favour the compliance cost minimization as the more natural approach, linking clearly to the individual banking decisions of LSEs. Letting $C(S_t, x_t)$ denote the compliance cost function at time t, then we have

$$C(S_t, \bar{x}_t) = \int_{\bar{x}_t(S_t)}^{\infty} f_t^{\text{SACP}}(u)du.$$

In order to obtain cleaner equations, we use $F_t(\bar{x}_t)$ to represent $\int_{\bar{x}_t}^{\infty} f_t^{\text{SACP}}(u)du$. Let Π be the set of all policies π i.e. functions that match each state S_t to a decision $X_t^\pi(S_t)$. We intend to find the best policy $\pi \in \Pi$, that which minimizes $\mathbb{E}_t \sum_{t'=t}^{T} e^{-r(t'-t)} C(S_{t'}, X_{t'}^\pi(S_{t'}))$:

$$V_t(S_t) = \min_{\pi \in \Pi} \mathbb{E}_t \sum_{t'=t}^{T} e^{-r(t'-t)} F_{t'}(X_{t'}^\pi(S_{t'})), \tag{3}$$

where $e^{-r(t'-t)}$ is the discount factor, and $\mathbb{E}_t$ is a time t conditional risk-neutral expectation.

Note that we are assuming that the competitive equilibrium can be modelled directly via the optimal decision of a single representative agent minimizing total costs. Much theory exists relating such a decision to that of individual agents able to trade with each other in the market, for example in the carbon market setting of Carmona *et al.* (2010) or Hitzemann and Uhrig-Homburg (2014), or for capacity expansion in power markets in Ehrenmann and Smeers (2011), so we choose to avoid adding an additional lengthy technical justification of this link here. Furthermore, we comment that an extension of the model to incorporate other costs and decisions is possible. While the marginal cost of SREC generation from existing solar fields is essentially zero, the upfront cost of installing new solar could of course be modelled, effectively adding an additional decision variable (and greater computational burden) to the problem. Instead, we choose to focus solely on the submission (or banking) decision, and assume the construction of new solar follows (2), a simple price-feedback relationship justified by empirical evidence in Coulon *et al.* (2015). Furthermore, one might argue that some measure of risk-aversion is important for both the banking and new investment decisions (given our premise that excess volatility discourages investment), but we leave these additional challenges for future work, simplifying the model sufficiently to facilitate the dynamic programming approach and to gain insight into the key implications of the ADAPT policy proposal.

5 Model Properties

In this section, we derive a few properties of the model introduced in the previous section that help us in analysing the market and solving the associated dynamic programming problem. We start by proving some features of the SREC markets that help us reduce the complexity of the stochastic dynamic program. Then, through a series of theorems and propositions, we prove different properties of the collective market behaviour. We use these results to obtain the optimal SREC submission policy representing the market behaviour for different market states. We then prove that under certain (typical) conditions there is no price difference among different vintage years, which is one of the reasons why the ADAPT policy promotes greater price certainty for investors. We also prove a theorem to compare total penalty payment under the two regimes. Note that we assume $\lambda > 0$, since the optimal policy under the classical market design is trivial (with non-increasing penalties), namely $\bar{x}_t = \min(\bar{b}_t, R_t)$, submit all you can up to the 'cliff' and bank everything beyond.

Our first step in analysing the ADAPT market design is to reduce the number of decision variables to one variable. The following theorem shows that if we only solve the problem for the total number of banked SRECs, $\bar{x}_t$, we can obtain the individual vintages, $x_{t,y}$, from $\bar{x}_t$.

Lemma 5.1 *If the optimal number of submitted SRECs at time t, $\bar{x}_t$, is known, the optimal number of SRECs submitted from different vintages at time t is given by*

$$x_{t,y} = \begin{cases} 0 & y < \lceil t \rceil - \tau \\ \min\{b_{t,y}, \bar{x}_t - \sum_{u=\lceil t \rceil - \tau}^{y-1} x_{t,u}\} & \lceil t \rceil - \tau \le y \le \lceil t \rceil \\ 0 & y > \lceil t \rceil \end{cases}$$

Proof Proof.

On one hand, all SREC vintages have the same impact on reducing the cost function $F_t(x_t)$ at each time (if they have not expired). On the other hand, the newer SRECs can be used further in the future to minimize $\mathbb{E}_t \sum_u e^{-r(u-t)} F_u(\bar{x}_u)$. Therefore, for any y, if $x_{t,y} < b_{t,y}$, $x_{t,y+1} = 0$ (i.e. the older SRECs must be submitted first). This means that the oldest SRECs ($y = \lceil t \rceil - \tau$) must all be submitted ($x_{t,\lceil t \rceil - \tau} = b_{t,\lceil t \rceil - \tau}$) unless this value exceeds $\bar{x}_t$. Therefore, we have $x_{t,\lceil t \rceil - \tau} = \min\{b_{t,\lceil t \rceil - \tau}, \bar{x}_t\}$. Similarly, the second oldest SRECs must all be submitted if this does not exceed the total remaining SRECs from the optimal submission $\bar{x}_t$, or $x_{t,\lceil t \rceil - \tau + 1} = \min\{b_{t,\lceil t \rceil - \tau + 1}, \bar{x}_t - x_{t,\lceil t \rceil - \tau}\}$. Extending this to other vintages $\lceil t \rceil - \tau \le y \le \lceil t \rceil$, we obtain $x_{t,y} = \min\{b_{t,y}, \bar{x}_t - \sum_{u=\lceil t \rceil - \tau}^{y-1} x_{t,u}\}$. The expired SRECs ($y < \lceil t \rceil - \tau$) and those not yet produced ($y > \lceil t \rceil$) of course cannot form a part of the optimal submitted SRECs $\bar{x}_t$. □ ∎

The simple and intuitive result above can be described as a 'first in first out' (FIFO) warehousing rule, as would apply to any perishable commodity. While it crucially allows us to reduce the decision variable to a scalar $\bar{x}_t$, the objective function (total discounted expected future costs), is still a function of the full vector $(b_{t,y})_y$ contained in S_t, which can lead to cases where different vintage SRECs have different values. Despite this warehousing concept, an SREC is a financial certificate, and does not have any storage or delivery costs or other constraints related to physical commodities. As such, standard no arbitrage arguments from finance apply, and SRECs must satisfy the martingale condition under the risk-neutral measure (at all times including compliance dates, as long as some are banked). At a compliance date non-expired SRECs must be worth at least as much as the current penalty rate paid per SREC, and at optimality would be banked if expected future prices are higher. Expired SRECs have no value.

Definition The price of SRECs of vintage y at time t satisfies

$$p_{t,y} = \begin{cases} \max\{f_t^{\text{SACP}}(x_t), e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y}\}\}, & t \leq y+\tau, \\ 0, & t > y+\tau. \end{cases} \tag{4}$$

Note that $f_t^{\text{SACP}}(x_t) = 0$ for times other than the compliance time ($t \notin \mathbb{N}$).

We note that the pricing equation closely resembles that of an American (or Bermudan) option, which is very natural since SRECs are used ('exercised') at predetermined compliance dates, or else expire. SREC prices in a competitive equilibrium can also be understood as the marginal benefit of having an additional unit of that vintage (or the marginal cost of having one less). Unlike for the discontinuous cliff policy, in the ADAPT policy (with $\lambda > 0$) we can equivalently use the derivative of the total cost function, as described by the proposition below.

Proposition 5.2 *The price of SRECs of vintage y at time t can be written as*

$$p_{t,y} = \begin{cases} -\frac{\partial V(S_t)}{\partial b_{t,y}}, & t \leq y+\tau, \\ 0, & t > y+\tau. \end{cases} \tag{5}$$

Proof Proof. Writing the objective function in (3) as

$$V_t(S_t) = \min_{\pi\in\Pi}\left\{F_t(X_t^\pi(S_t)) + e^{-r\Delta t}\mathbb{E}_t\sum_{t'=t+\Delta t}^{T} e^{-r(t'-(t+\Delta t))}F_{t'}(X_{t'}^\pi(S_{t'}))\right\},$$

we can see that an additional SREC will either be used to reduce the first term or the second (depending on which is optimal). Therefore, differentiating with respect to $b_{t,y}$ (and noting that vintage y is worthless after $y+\tau$) returns precisely the price definition equation in (4). □ ▮

Next, to understand the optimal policy for $\bar{x}_t$, we first need to prove the following lemmas. We assume $\tau > 0$ throughout, since otherwise there is no banking decision to make. The first lemma describes the required price for any SREC vintages used for compliance.

Lemma 5.3 *Let $\bar{x}_t = X_t^*(S_t)$ represent the optimal policy at any compliance time $t \in \mathbb{N}$. For any (and all) vintage y such that $x_{t,y} > 0$, we must have $p_{t,y} = f^{SACP}(\bar{x}_t)$.*

Proof Proof.

From (4), we know that $p_{t,y} \geq f^{\text{SACP}}(\bar{x}_t)$. Suppose that $p_{t,y} > f^{\text{SACP}}(\bar{x}_t)$. Since we have $p_{t,y} = -\frac{\partial V(S_t)}{\partial b_{t,y}}$ by Proposition 1, we know an additional SREC (of vintage y) can reduce our objective function by $p_{t,y}$. Therefore it cannot be optimal to have submitted an SREC at SACP level $f^{\text{SACP}}(\bar{x}_t)$, hence contradicting $x_{t,y} > 0$. (Note also that intuitively, if the market price is higher than the SACP value, it's better to sell the SREC in the market and pay a slightly higher penalty than to submit it for compliance. This is not always true for the discontinuous cliff policy.) $\square$ ∎

The next lemma covers the situation between compliance dates and that of any SREC vintages not used for compliance.

Lemma 5.4 *For any (and all) vintage y such that $x_{t,y} = 0$, we must have $p_{t,y} = e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y}\}$.*

Proof Proof.

The value function can be written

$$V_t(S_t) = \min_{\pi \in \Pi} \left\{ F_t(X_t^\pi(S_t)) + e^{-r\Delta t}\mathbb{E}_t \sum_{t'=t+\Delta t}^{T} e^{-r(t'-(t+\Delta t))} F_{t'}(X_{t'}^\pi(S_{t'})) \right\}.$$

Since $x_{t,y} = 0$, we know that $F_t(X_t^\pi(S_t))$ is not a function of $b_{t,y}$, and hence

$$\frac{\partial V_t(S_t)}{\partial b_{t,y}} = \mathrm{e}^{-r\Delta t}\frac{\partial}{\partial b_{t,y}} \min_{\pi \in \Pi} \mathbb{E}_t \sum_{t'=t+\Delta t}^{T} e^{-r(t'-t)} F_{t'}(X_{t'}^\pi(S_{t'})) = \mathrm{e}^{-r\Delta t}\frac{\partial V_{t+\Delta t}(S_{t+\Delta t})}{\partial b_{t,y}}$$

Thus, by Proposition 1, $p_{t,y} = \mathrm{e}^{-r\Delta t}\mathbb{E}_t\left[p_{t+\Delta t,y}\right]$. $\square$ ∎

Finally, consider the case of an SREC vintage reaching its final expiry.

Lemma 5.5 *Let $\bar{x}_t = X_t^*(S_t)$ represent the optimal policy at any time t. At the end of a vintage's life (at $t = y + \tau$ for vintage y), if $b_{t,y} > 0$, then $x_{t,y} = b_{t,y}$ and $p_{t,y} = f^{SACP}(\bar{x}_t)$.*

Proof Proof.

By definition, $p_{t+\Delta t,y} = 0$ at $t = y + \tau$, so any remaining SRECs will expire worthless if not used at t. Since F is strictly non-increasing in $\bar{x}_t$, submitting additional SRECs can only reduce one's costs, so $x_{t,y} = b_{t,y}$ must be within the optimal set (although any submission beyond $(1+\lambda)R_t$ would no longer have any impact on the objective function). $\square$ ∎

The result above clearly implies that at any $t \in \mathbb{N}$, the optimal submission decision lies in the range $\bar{x}_t \in [b_{t,t-\tau}, \bar{b}_t]$. We can now prove the main theorem describing how price dynamics are linked to submission and banking decisions, starting directly from the optimization problem.

Theorem 5.6 *Let $\bar{x}_t = X_t^*(S_t)$ represent the optimal policy at any compliance time $t \in \mathbb{N}$. Then*

$$f^{SACP}(\bar{x}_t) = \max\left(f^{SACP}(\bar{b}_t), \min\left(f^{SACP}(b_{t,t-\tau}), e^{-r\Delta t}\mathbb{E}_t\left[p_{t+\Delta t,y^\star}\right]\right)\right),$$

where $y^\star = \max\{y : x_{t,y} > 0\}$.

Proof Proof. Letting b_t, x_t and g_t be vectors (where for g_t only current vintage can be non-zero),

$$
\begin{aligned}
V_t(b_t, \cdot) &= \min_{\pi \in \Pi} \mathbb{E}_t \sum_{t'=t+\Delta t}^{T} e^{-r(t'-(t+\Delta t))} F_{t'}(X_{t'}^{\pi}(S_{t'})) && (6) \\
&= \min_{\bar{x}_t \in [b_{t,t-\tau}, \bar{b}_t]} \left\{ F_t(\bar{x}_t) + \mathrm{e}^{-r\Delta t} \mathbb{E}_t \left[V_{t+\Delta t}(b_t - x_t + g_t \Delta t, \cdot) \right] . \right\} && (7)
\end{aligned}
$$

Now we look at the Karush-Kuhn-Tucker (KKT) conditions of this constrained maximization (with two constraints, so we introduce KKT multipliers μ_1 and μ_2). Since $\frac{\mathrm{d}F(x)}{\mathrm{d}\bar{x}} = -f^{\mathrm{SACP}}(\bar{x})$, the stationarity condition (with optimal solution denoted by $\bar{x}^{\star}$) gives:

$$
-f^{\mathrm{SACP}}(\bar{x}_t^{\star}) + \mathrm{e}^{-r\Delta t} \frac{\partial}{\partial \bar{x}} \mathbb{E}_t \left[V_{t+\Delta t}(b_t - x_t + g_t \Delta t, \cdot) \right] \big|_{\bar{x} = \bar{x}^{\star}} = \mu_1 - \mu_2
$$

while the complementary slackness conditions give

$$
\mu_1(\bar{x}^{\star} - \bar{b}_t) = 0 \qquad \text{and} \qquad \mu_2(b_{t,t-\tau} - \bar{x}^{\star}) = 0.
$$

Also, $b_{t,t-\tau} \leq \bar{x}^{\star} \leq \bar{b}_t$ and $\mu_1, \mu_2 \geq 0$. Together these imply that if $b_{t,t-\tau} < \bar{x}^{\star} < \bar{b}_t$, then we must have $\mu_1 = \mu_2 = 0$, so

$$
-f^{\mathrm{SACP}}(\bar{x}_t^{\star}) + \mathrm{e}^{-r\Delta t} \frac{\partial}{\partial \bar{x}} \mathbb{E}_t \left[V_{t+\Delta t}(b_t - x_t + g_t \Delta t, \cdot) \right] \big|_{\bar{x} = \bar{x}^{\star}} = 0
$$

or equivalently, since Lemma 1 tells us that derivatives with respect to $\bar{x}$ correspond to derivatives with respect to the most recent vintage used for submission,

$$
f^{\mathrm{SACP}}(\bar{x}_t^{\star}) = -\mathrm{e}^{-r\Delta t} \mathbb{E}_t \left[\frac{\partial}{\partial x_{t,y}^{\star}} V_{t+\Delta t}(b_t - x_t + g_t \Delta t, \cdot) \right]
$$

where $y^{\star} = \max\{y : x_{t,y}^{\star} > 0\}$. Thus, if $b_{t,t-\tau} < \bar{x}^{\star} < \bar{b}_t$, then using Proposition 1 and Lemma 2 for any submitted vintage y (i.e. $x_{t,y} > 0$),

$$
p_{t,y} = \mathrm{e}^{-r\Delta t} \mathbb{E}_t \left[p_{t+\Delta t, y^{\star}} \right].
$$

Overall, for any $t \in \mathbb{N}$, we must have

$$
(x_t^{\star} - \bar{b}_t)(x^{\star} - b_{t,t-\tau}) \left(p_{t,y} - \mathrm{e}^{-r\Delta t} \mathbb{E}_t \left[p_{t+\Delta t, y^{\star}} \right] \right) = 0
$$

and since $f^{\mathrm{SACP}}(\cdot)$ is a strictly non-increasing function, we can also say that

$$
f^{\mathrm{SACP}}(\bar{x}_t^{\star}) = \max \left(f^{\mathrm{SACP}}(\bar{b}_t), \min \left(f^{\mathrm{SACP}}(b_{t,t-\tau}), \mathrm{e}^{-r\Delta t} \mathbb{E}_t \left[p_{t+\Delta t, y^{\star}} \right] \right) \right).
$$

□ ∎

In the typical case of decreasing penalties over time, we can also link the optimal policy $\bar{x}_t$ with that of the following compliance date, $\bar{x}_{t+1}$:

Corollary 5.7 *Let $\bar{x}_t = X_t^*(S_t)$ represent the optimal policy at any $t \in \mathbb{N}$. Suppose that penalties P_y are non-increasing in y (as is typical), and that $R_y > 0$ for all y. Then,*

$$f^{SACP}(\bar{x}_t) = \max\left(f^{SACP}(\bar{b}_t), \min\left(f^{SACP}(b_{t,t-\tau}), e^{-r}\mathbb{E}_t\left[f^{SACP}(\bar{x}_{t+1})\right]\right)\right).$$

Proof Proof.

To extend the theorem above to the corollary, we need to show that

$$\mathrm{e}^{-r\Delta t}\mathbb{E}_t\left[p_{t+\Delta t,y^\star}\right] = \mathrm{e}^{-r}\mathbb{E}_t\left[f^{\mathrm{SACP}}(\bar{x}_{t+1})\right],$$

where $y^\star = \max\{y : x^\star_{t,y} > 0\}$. First note that $\mathrm{e}^{-r\Delta t}\mathbb{E}_t\left[p_{t+\Delta t,y^\star}\right] = \mathrm{e}^{-r}\mathbb{E}_t\left[p_{t+1,y^\star}\right]$ by Lemma 3 since $\bar{x}_t = 0$ for $t \notin \mathbb{N}$ (i.e. the martingale condition holds between compliance dates).

Now there are two cases to consider:

(i) $x_{t,y^\star} < b_{t,y^\star}$: Since $R_{t+1} > 0$ and $P_{t+1} \geq P_u \, \forall u > t+1$ by assumption, we can deduce that $x_{t+1,y^\star} > 0$ (i.e. at least some of the remaining SRECs of vintage $y^\star$ will be used for compliance), as we know the first of these will reduce the total cost function by P_{t+1}, a larger number than possible in any future year. Thus, by Lemma 1, $p_{t+1,y^\star} = f^{\mathrm{SACP}}(\bar{x}_{t+1})$.

(ii) $x_{t,y^\star} = b_{t,y^\star}$: As no SRECs of vintage $y^\star$ remain beyond time t, $p_{t+1,y^\star}$ is a hypothetical price in the event of suboptimal banking. Nonetheless, we can still claim that $p_{t+1,y^\star} = f^{\mathrm{SACP}}(\bar{x}_{t+1})$ must hold by Lemma 1 in such a case. □ ∎

The theorem and corollary above describe all three possible cases at a submission time $t \in \mathbb{N}$: undersupply, (severe) oversupply, and normal conditions. In the first case, $\bar{x}_t = \bar{b}_t$ (since $f^{\mathrm{SACP}}(\bar{x}_t) = f^{\mathrm{SACP}}(\bar{b}_t)$), so all available SRECs are submitted in order to reduce penalties as much as possible. In the second case, only the oldest vintage is submitted ($x_{t,y} = b_{t,y}$) in order to avoid SRECs expiring worthless. Given $\tau = 4$ as in New Jersey, this oversupply case is particularly extreme and unlikely, as it would imply that all five vintages still exist in the market, and moreover that $b_{t,t-4}$ is a large enough number that it would not be optimal to use any of $b_{t,t-3}$. Finally, the most typical case of $b_{t,t-\tau} < \bar{x}^\star < \bar{b}_t$ is between these extremes, with some SRECs submitted and others saved in order to balance with their expected future value.

In all three cases, Lemma 2 tells us that $p_{t,y} = f^{\mathrm{SACP}}(\bar{x}_t)$ for all vintages y with $x_{t,y} > 0$. In the undersupply case ($p_{t,y} = f^{\mathrm{SACP}}(\bar{b}_t)$ for all vintages), all existing SRECs are submitted, so $\mathbb{E}_t[p_{t+\Delta t,y^\star}]$ arguably does not exist, since none of the current vintages will trade next period. However, if (suboptimally) any SRECs remained in the market, their future price would drop. In the extreme oversupply case ($p_{t,t-\tau} = f^{\mathrm{SACP}}(b_{t,t-\tau})$ for the only submitted vintage), other vintages may have higher prices given by their continuation value, as in Corollary 1. Finally, in the typical case, all vintages $y \geq y^\star$ will have prices equal to $\mathrm{e}^{-r\Delta t}\mathbb{E}_t\left[p_{t+\Delta t,y^\star}\right]$, the continuation value of the newest vintage submitted. It is theoretically possible that other vintages $y > y^\star$ exist and have higher prices. Another representation of the price $p_{t,y}$ can help provide further intuition, showing that the value of the SREC is the maximum over its compliance cost impact at all future exercise opportunities for which $x_{t,y} > 0$ optimally. We express this through the following proposition.

Proposition 5.8 *Let $\bar{x}_t = X_t^*(S_t)$ represent the optimal policy at any compliance time $t \in \mathbb{N}$.*

The price of SRECs of vintage y at time t can be written

$$p_{t,y} = \max_{u \in \{\lceil t \rceil, \lceil t \rceil + 1, \ldots, \lceil t \rceil + \tau\}} \mathbb{E}_t \left[e^{-r(u-t)} f^{SACP}(\bar{x}_u) 1_{\{x_{u,y} > 0\}} \right].$$

Proof Proof.

For $t \in (y + \tau - 1, y + \tau]$, we have no further banking decisions remaining for vintage y, and from Lemmas 3 and 4, we must have

$$p_{t,y} = e^{-r(y+\tau-t)} \mathbb{E}_t \left[f^{\mathrm{SACP}}(\bar{x}_{y+\tau}) \right].$$

At $t = y + \tau - 1$, using Corollary 1 and Lemmas 2 and 3, we then have:

$$p_{t,y} = \begin{cases} f^{\mathrm{SACP}}(\bar{x}_t), & \text{if } x_{t,y} > 0 \\ e^{-r} \mathbb{E}_t \left[f^{\mathrm{SACP}}(\bar{x}_{y+\tau}) \right] \geq f^{\mathrm{SACP}}(\bar{x}_t), & \text{if } x_{t,y} = 0. \end{cases}$$

By repeating this argument iteratively back in time we obtain the result. □ ∎

The possibility of different prices for different vintages introduces a high dimensional state variable. However, investigations of realistic scenarios reveal that it is very likely for all vintages to have the same price, allowing us to approximate the problem by a much lower dimensional one with $\bar{b}_t$ as state variable. Using Proposition 2, we can show that for neighbouring vintages (y and $y+1$ say), $p_{t,y} < p_{t,y+1}$ is possible if and only if there exists some positive probability that we have an excess of vintage y SRECs to dispose of at their expiry.

Theorem 5.9 *Let $\bar{x}_t = X_t^*(S_t)$ be the optimal policy, and $\mathbb{P}_t$ the conditional probability at t. If*

$$\mathbb{P}_t \left\{ \bar{x}_u = b_{u,u-\tau} > 0 \right\} = 0, \quad \forall u \in \{\lceil t \rceil, \lceil t \rceil + 1, \ldots, \lceil t \rceil + \tau\},$$

then all SREC vintages have equal prices at time t. (i.e. $p_{t,u} = p_{t,v}, \forall u, v \in \{\lceil t \rceil, \ldots, \lceil t \rceil - \tau\}$).

Proof Proof. Assume $P\left\{ \bar{x}_u = b_{u,u-\tau} > 0 \right\} = 0, \quad \forall u \in \{\lceil t \rceil, \lceil t \rceil + 1, \ldots, \lceil t \rceil + \tau\}$. Now suppose (for a contradiction) that $p_{t,y} < p_{t,y+1}$ for some y and $y+1$ in $\{\lceil t \rceil, \ldots, \lceil t \rceil - \tau\}$, noting that $p_{t,y} > p_{t,y+1}$ is precluded anyway by no arbitrage since newer vintages give all the same benefits as older ones plus more, the theorem is a simple extension of this claim.

From Proposition 2, we can deduce that either there must exist some paths $A \in \Omega$ with $P\{A\} > 0$ such that for some $\tilde{u} \in \{\lceil t \rceil, \lceil t \rceil + 1, \ldots, y + \tau + 1\}$ we have $x_{\tilde{u},y} = 0, x_{\tilde{u},y+1} > 0$ and $p_{\tilde{u},y} < p_{\tilde{u},y+1}$. However, if $\tilde{u} \leq y + \tau$ (i.e., before vintage y expires), we can show a contradiction: firstly if $b_{u,y} > 0$ for $A \in \Omega$, then $x_{\tilde{u},y} = 0, x_{\tilde{u},y+1} > 0$ contradicts Lemma 1; alternatively, if $b_{u,y} = 0$, then the 'hypothetical' price $p_{\tilde{u},y} = f^{\mathrm{SACP}}(\bar{x}_{\tilde{u}})$ still holds, which equals $p_{\tilde{u},y+1}$, again a contradiction.

This proves that no price can stem from the earlier compliance dates and we must have $\tilde{u} = y + \tau + 1$, for which $0 = p_{\tilde{u},y} < p_{\tilde{u},y+1}$ for paths $A \in \Omega$, since vintage y has expired. However this is not necessarily sufficient to lead to $p_{t,y} < p_{t,y+1}$. We know from Lemma 4 that at time $y + \tau$, $\bar{x}_{y+\tau} \geq b_{y+\tau,y}$. Suppose that $\bar{x}_{y+\tau} > b_{y+\tau,y}$. Then by Lemma 1, $x_{y+\tau,y+1} > 0$ and $p_{y+\tau,y} = p_{y+\tau,y+1}$, again contradicting the assumed price difference between vintages. Therefore the only remaining possibility is that $\bar{x}_{y+\tau} = b_{y+\tau,y}$ (and $b_{y+\tau,y} > 0$) for paths $A \in \Omega$, in which case from Corollary 2, we can have $p_{y+\tau,y} = f^{\text{SACP}}(\bar{x}_{y+\tau}) < \mathrm{e}^{-r}\mathbb{E}_{y+\tau}\left[f^{\text{SACP}}(\bar{x}_{y+\tau+1})\right] = p_{y+\tau,y+1}$. This event was ruled out at the start of our proof in order to obtain this contradiction, which completes the proof of our claim. The extension of the claim from neighbouring vintages y and $y+1$ to any non-expired vintages is trivial. $\square$ $\blacksquare$

Note also that the inclusion of $b_{y+\tau,y} > 0$ in the theorem above is not needed in the (typical) case of non-increasing penalties P_t, since it only serves to eliminate the scenario where $\bar{x}_t = 0$ at $t \in \mathbb{N}$ despite $\bar{b}_t > 0$.

A simpler sufficient (far from necessary) condition for price convergence across vintages is

$$\bar{x}_t > \sum_{y=t-\tau}^{t-1} b_{t,y}, \qquad \forall t \in \mathbb{N}$$

as it then guarantees that $x_{t,t} > 0$ (i.e., at least some of the newest vintage SRECs are submitted). Recalling Figure 1b, we note that at the peak of oversupply in the New Jersey market (2012-13), banked certificates still remained between 40% and 50% of the following year's requirement, suggesting that the newest vintage would always be needed for compliance, easily enough to equalize prices, even with a significantly positive λ under ADAPT (since with non-increasing penalties it is always optimal to submit enough SRECs to reach the beginning of the slope at least). It is important to recall that the natural feedback effect in the market naturally serves to avoid extreme imbalances since new generation slows during oversupply, and accelerates during undersupply.

On the other hand, price differences between vintages are fairly common for the cliff policy (e.g., Figure 1a) so it is not immediately intuitive how the sloped policy eliminates such differences in all but extreme scenarios. We provide an illustrative toy example below:

Example 5.10 *Let $r = 0$. Suppose also we have no randomness in the model, and perfect foresight on future SREC supply. Set $\tau = 1$ so that we have (at most) two vintages available at any time. Vintage 0 expires in year 1, and vintage 1 expires in year 2. Requirements are $R_1 = R_2 = 10,000$, and penalties $P_1 = P_2 = P$. The banked certificates available by the end of year 1 are*

$$b_{1,0} = 8,000, \quad b_{1,1} = 6,000$$

and we know that next year we will also have some vintage 2 SRECs available:

$$b_{2,2} = 5,000$$

We consider the optimal submission decisions for year 1 and year 2 to minimize total cost, firstly under the case of the cliff policy, and secondly for ADAPT's sloped policy (with $\lambda = 0.1$).

1. Cliff Policy: *In Year 1, 14,000 SRECs are available, so 10,000 can be used to meet the first requirement, with 4 banked to Year 2. Since no penalty is paid, $p_{1,0} = 0$. Then only 9,000 will be available for Year 2 compliance, so the requirement will be missed, and $p_{1,1} = P$, the second year's penalty. The total amount of penalty paid for non-compliance (over the two years) is $1,000P$.*

2. ADAPT Policy ($\lambda = 0.1$): *Suppose we choose to submit* $(1+\lambda)R_1$ *in Year 1 to reduce our penalty to zero. Then we would submit 11,000 in Year 1, leaving only 8,000 for Year 2 and a total penalty of* $2,000P$ *(the area* $\int_{8000}^{11000} f^{SACP}(x)$*). Prices are again* $p_{1,0} = 0, p_{1,1} = P$ *in this (suboptimal) case. Instead, the optimal policy (ignoring Year 3 and beyond) is to spread our compliance costs between years, by submitting 9,500 SRECs each of the two years. The two vintage prices are then equal at* $p_{1,0} = p_{1,1} = 0.75P$*, and the total penalty paid each year is* $1,500(0.75P)/2$*, giving a combined value of* $1,125P$*.*

While the total penalty of $1,125P$ *under ADAPT is slightly higher than the* $1,000P$ *under the cliff policy, it is significantly lower than the* $2,000P$ *under the naive policy of lowering the first year's penalty to zero at the expense of the second. We clearly see the incentive for market participants to balance their banking decisions with future price expectations, and the resulting equalization of vintage prices. Note that in a more complete example, the feedback effect should also be included such that the future generation of SRECs (*$b_{2,2}$*, say) should change as current prices (linked to current decisions) change. Similarly, with* $\alpha > 0$ *in the ADAPT policy, the future requirement would also respond to earlier decisions. However, although numbers would change slightly, both of these effects would not change the overall features of the comparison being made above, so we have ignored them for simplicity in this example.*

Note also that the inventory of SRECs in the simple example above is rather unrealistic, as there are more of the older vintages remaining than the newer ones, the reverse of what one would expect (both because of Lemma 1 and because of the growth of generation in most markets). Even still, prices across vintages are equal. One would have to change the balance to $b_{1,0} = 9,501, b_{1,1} = 4,499$ *before a price difference would emerge, because it would no longer be optimal to only submit 9,500 in year 1 (and waste one SREC).*

In other words, a price difference can only emerge if the optimal submission decision x_t, needed to balance today's penalty with expected future prices, would fall below the bound $b_{t,t-\tau}$ either this period or with some probability in a future period. In such a case, it is instead preferable to submit $b_{t,t-\tau}$ to avoid wasting SRECs, thus bringing down the price of the expiring vintage compared to newer ones. While the balancing of prices across time tends to spread penalties more evenly under ADAPT, the example above illustrates that the total expected penalty payment under both regimes does not remain the same. The following theorem shows that the total penalty payment is always more under the sloped market design.

Theorem 5.11 *Total penalty payment of the sloped market design is always greater than or equal to that paid under the original step mechanism.*

Proof Proof. Let $\bar{x}_t$ denote the total submitted SRECs at time $t \in \mathbb{N}$ under the optimal policy for the ADAPT compliance regime. Let $F_t^{\text{ADAPT}}(\bar{x}_t)$ and $F_t^{\text{Cliff}}(\bar{x}_t)$ represent the total compliance costs at each time t under each regime. Consider the two possible cases for $\bar{x}_t$, for any $t \in \mathbb{N}$: (i) If $\bar{x}_t \geq R_t$, we have

$$F_t^{\text{Cliff}}(\bar{x}_t) = \int_{\bar{x}_t}^{\infty} f_t^{\text{Cliff}}(z)dz = 0 \leq \int_{x_t}^{\infty} f_t^{\text{ADAPT}}(z)dz = F_t^{\text{ADAPT}}(\bar{x}_t).$$

(ii) If $0 \leq \bar{x}_t \leq R_t$, we have $f_t^{\text{Cliff}}(\bar{x}_t) \geq f_t^{\text{ADAPT}}(\bar{x}_t)$. Since by construction the total areas under the two penalty functions are equal ($\int_0^\infty f_t^{\text{Cliff}}(z)dz = \int_0^\infty f_t^{\text{ADAPT}}(z)dz$), we conclude

$$F_t^{\text{Cliff}}(\bar{x}_t) = \int_{\bar{x}_t}^{\infty} f_t^{\text{Cliff}}(z)dz \leq \int_{x_t}^{\infty} f_t^{\text{ADAPT}}(z)dz = F_t^{\text{ADAPT}}(\bar{x}_t).$$

Finally, let $\tilde{x}_t$, be the optimal policy for the classical step regime. By definition, we must have

$$F_t^{\text{Cliff}}(\tilde{x}_t) \leq F_t^{\text{Cliff}}(\bar{x}_t).$$

Hence as required,

$$F_t^{\text{Cliff}}(\tilde{x}_t) \leq F_t^{\text{ADAPT}}(\bar{x}_t).$$

□ ∎

6 Algorithm for Solving the Model

The original dynamic programming model is computationally intractable due to the dimensionality of the state variable. However, we can reduce the dimensionality, preserving the structure and behavior of the problem yet producing a model that can be solved exactly.

Specifically, the number of dimensions can be reduced greatly by using the scalar $\bar{b}_t$, giving the aggregate number of banked SRECs, instead of the vector $(b_{t,y})_y$ which captures the banking by vintage. The vintage-specific values appear in the result

$$p_{t,y} = \max\left(f^{\text{SACP}}(\bar{b}_t), \min\left(f^{\text{SACP}}(b_{t,t-\tau}), \mathrm{e}^{-r\Delta t}\mathbb{E}_t\left[p_{t+\Delta t, y^\star}\right]\right)\right), \quad \forall y \text{ such that } x_{t,y} > 0,$$

where $y^\star = \max\{y : x_{t,y} > 0\}$, but are only relevant if the number of older SRECs that are about to expire is very large at some $t \in \mathbb{N}$. This is quite unlikely to happen in reality (in fact, for $\tau = 4$, $b_{t,t-\tau} = 0$ is virtually guaranteed), specifically because the oldest SRECs are the first to be submitted at each compliance time (by Lemma 1). Therefore, when generating our price surfaces, we find the value function in terms of the scalar $\bar{b}_t$, with transition function given by

$$\bar{b}_{t+\Delta t} = \bar{b}_t + g_t \Delta t - \bar{x}_t.$$

However, as we simulate forward, we retain the vector of banked SRECs by vintage, $(b_{t,y})_y$, making it possible to check to see if our dimension reduction is valid. Numerical experiments with $\tau = 4$ and our chosen paremeters (see next section) reveal no contradiction of the assumption.

Assuming penalties P_y are non-increasing and that $P\{\bar{x}_t = b_{t,t-\tau}\} = 0$ (as in Theorem 2), we have by Corollary 2 that $f_t^{\text{SACP}}(\bar{x}_t) = \max\{f_t^{\text{SACP}}(\bar{b}_t), e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(\bar{x}_{t+1})\}$ and by Theorem 1,

$$f_t^{\text{SACP}}(\bar{x}_t) = \max\{f_t^{\text{SACP}}(\bar{b}_t), e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t, y_t^\star}\}\}, \tag{8}$$

where $y^\star = \max\{y : x_{t,y} > 0\}$. Given our dimension reduction, $y^\star$ is not observable when solving for the price of some vintage y. However, Theorem 2 allows us to replace $y^\star$ by y. Thus, the optimal x_t can be obtained from

$$\bar{x}_t = \min\{\bar{b}_t, (f^{\text{SACP}})_t^{-1}(e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y_t}\})\} \tag{9}$$

Note that because P_t is non-increasing in time, at any time $t \in \mathbb{N}$ the expected future price is no larger than the maximum penalty at time t $\left(\mathbb{E}_t\{p_{t+\Delta t,y_t^{max}}\} \leq P_t\right)$. Thus, $e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y_t^{max}}\} \leq \mathbb{E}_t\{p_{t+\Delta t,y_t^{max}}\}$ is in the sloped area of f_t^{SACP} and therefore $(f^{\text{SACP}})_t^{-1}$ is defined.

This means we can solve our dynamic program based on SREC prices instead of the original objective function (i.e. social welfare). This simplifies calculations at each time step and directly outputs SREC prices. According to the definition of SREC prices (equation (4)), we obtain,

$$p_{t,y} = \exp(-r\Delta t)\mathbb{E}_t[p_{t+\Delta t,y}], \qquad t \notin \mathbb{N}, \tag{10}$$

and at the compliance dates ($t \in \mathbb{N}$), we have

$$p_{t,y} = \max\{f_t^{\text{SACP}}(\bar{x}_t), \exp(-r\Delta t)\mathbb{E}_t[p_{t+\Delta t,y}]\}, \qquad t \in \mathbb{N}. \tag{11}$$

We can use equations (9), (10), and (11) to compute the SREC prices using backward induction on a discretized state space. We solve the dynamic program for each vintage in turn, using differents grids and starting with the newest vintage y^{max} each year due to its assumed role in driving the price feedback. We compute the price surfaces according to the following algorithm.

1. Discretize the state variable $S_t = (b_t, \hat{g}_t, s_t)$ on a grid ranging over $[0, \kappa^b R_{y+\tau}]$, $[0, \kappa^g R_{y+\tau}]$, and $[-\kappa^s R_{y+\tau}, \kappa^s R_{y+\tau}]$ respectively for each vintage year y, where $\kappa^b, \kappa^g > 1$ (e.g. 1.5), and $\kappa^s > 0$ (e.g. 0.5). Also discretize the distribution of the noise variable ε_t into n outcomes $\varepsilon_{t,i}$ with probability $pr\{\varepsilon_{t,i}\}$, for $i = 1, \ldots, n$.

2. Initialize the dynamic program by calculating $p_{T,y^{fin}} = f_T^{\text{SACP}}(\bar{b}_T)$ at all grid points, where y^{fin} denotes the final SREC vintage in our simulation, and $T = y^{fin} + \tau$.

3. Go backward through time and compute prices of all (possibly existing) SREC vintages $(y_t^{min}, \ldots, y_t^{max})$ from (10) and (11) at each grid point $(\bar{b}_t, \hat{g}_t, s_t)$:
   (a) If $t \notin \mathbb{N}$, $\tilde{x}_t = 0$, otherwise find $\tilde{x}_t$ by solving $f_t^{\text{SACP}}(\tilde{x}_t) = e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y_t^{max}}|\tilde{x}_t\}$:
      i. Set $\tilde{x}_t = (1+\lambda)R_t$; at this point $f_t^{\text{SACP}}(\tilde{x}_t) = 0$.
      ii. $x_t = \min\{\tilde{x}_t, \bar{b}_t\}$
      iii. $p_{t,y_t^{max}} = \max\{f_t^{\text{SACP}}(\tilde{x}_t), f_t^{\text{SACP}}(\bar{b}_t)\}$.
      iv. From (2) $\hat{g}_{t+\Delta t} = \hat{g}_t \exp(a_5\Delta t + a_6 p_{t,y^{max}}\Delta t)$;
      v. For each $i = 1, \ldots, n$ from (1)

$$\begin{aligned} g_{t+\Delta t,i} \quad = \quad & \hat{g}_{t+\Delta t}\exp\Big(a_1\sin(4\pi(t+\Delta t)) + a_2\cos(4\pi(t+\Delta t)) \\ & + a_3\sin(2\pi(t+\Delta t)) + a_4\cos(2\pi(t+\Delta t)) + \varepsilon_{t,i}\Big). \end{aligned}$$

      vi. $\bar{b}_{t+\Delta t,i} = \bar{b}_t + g_{t+\Delta t,i}\Delta t - x_t$.
      vii. $s_{t+\Delta t} = \bar{b}_t - \tilde{R}_t$.

a_0	a_1	a_2	a_3	a_4	a_5	a_6	σ
10.9558	-0.1209	0.0900	0.2151	0.3859	-0.0151	1.27×10^{-3}	0.186

Table 2: Estimated parameters for our linear model

viii. Using the discretized distribution function

$$\mathbb{E}_t[p_{t+\Delta t,y_t^{max}}] = \sum_{i=1}^{n} pr\{\varepsilon_{t,i}\} p_{t+\Delta t,y_t^{max}}\left(\bar{b}_{t+\Delta t,i}, \hat{g}_{t+\Delta t}, s_{t+\Delta t}\right).$$

ix. If $f_t^{\text{SACP}}(\tilde{x}_t) < e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y_{t+\Delta t}^{max}}\}$, update $\tilde{x}_t$ to $\tilde{x}_t - k$ with $k > 0$, and go to (ii).

(b) Set $\bar{x}_t = \min\{\bar{b}_t, \tilde{x}_t\}$, and compute $\bar{b}_{t+\Delta t}$, $\hat{g}_{t+\Delta t}$, and $s_{t+\Delta t}$ as of (a iii) to (a vi) equations.

(c) For $y : y_t^{max}$ to y_t^{min}

i. Calculate $\mathbb{E}_t[p_{t+\Delta t,y}] = \sum_{i=1}^{n} pr\{\varepsilon_{t,i}\} p_{t+\Delta t,y}\left(\bar{b}_{t+\Delta t}, \hat{g}_{t+\Delta t}, s_{t+\Delta t}\right)$.

ii. $p_{t,y} = \max\{f_t^{\text{SACP}}(\bar{x}_t), e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y}\}\}$.

CPU times for solving a simulation over a decade (with an acceptable level of accuracy), with a single fine-grained grid, are excessive (on the order of several weeks). As our state variables (e.g. generation) lie on different ranges for early and late years, we can use separate smaller grids for different vintage years (as given in step 1 above) instead of a single large grid. The grid size dynamically changes according to the requirement level. Going forward in time, we need to convert any point from an earlier year grid to the corresponding point in the newer year. This maintains the same level of accuracy, while keeping the problem computationally tractable. Experiments were run for 50 to 100 grid points for each state variable for each vintage year, and this was found to produce an acceptable tradeoff between accuracy and CPU times.

7 Experiments

To gain insight into the effects of different SREC policies, we must first choose some reasonable parameters for the current market design to serve as our benchmark. We can then modify some of these parameters to design new markets and assess their performance. To represent the current market mechanism, for parameters $\tilde{R}_t$ and P_t we use the values given in Table 1, which are based on the latest regulation changes in NJ. Also, we use a constant interest rate of 2% estimated based on historical interest rates (the interest rate has little effect on the results of our experiments). The current SREC market design is characterized by a sharp cliff and non-adaptive requirements, a case we compare to by letting $\lambda = 0, \alpha = 0$.

The parameters for the generation model (for g_t) are taken directly from Coulon *et al.* (2015) (given in Table 2), and are obtained by fitting a linear model to the log of the historical generation data (see Coulon *et al.* (2015) for more details). A normal distribution with mean zero and standard deviation 0.186 seems to characterize the noise ε_t well.

We now report on experiments designed to provide insights into the effect of different market designs. We begin our experiments assuming a steady growth in targets, a strategy that ensures that SREC prices do not rapidly fall towards zero. For this purpose, we assume that requirements are determined by $R_y = \alpha \exp \beta(y - 14)$, with α equal to the requirement level of EY2014 and β equal to a positive number ($\beta = 0.35$ in the following experiments). Also, we assume five years of lifetime ($\tau = 4$). After studying the behavior of market designs using this formula for requirements, we report on a study of the current requirement schedule.

7.1 The slope of the SACP function

There are four critical differences between ADAPT and the current market mechanism. Firstly, even for a small positive λ (producing a shallow SACP function), extreme prices near zero or P_t are rare, only happening if the total banked SRECs are either very high or very low. Secondly, in the sloped mechanism, penalties can have any values between 0 and P_t unlike the binary nature of the step mechanism. Thirdly, in the step market design, if the total banked SRECs are less than the requirement, they must all be submitted, as P_t is a decreasing sequence through time and so the current penalty is more than any possible future price. Thus, in effect there is no real choice for how many SRECs to submit and how many to bank in the step mechanism, while under ADAPT participants can choose the submission value to balance prices before and after submission (to minimize compliance costs). Finally, this additonal flexibility results in equal prices across vintages under ADAPT.

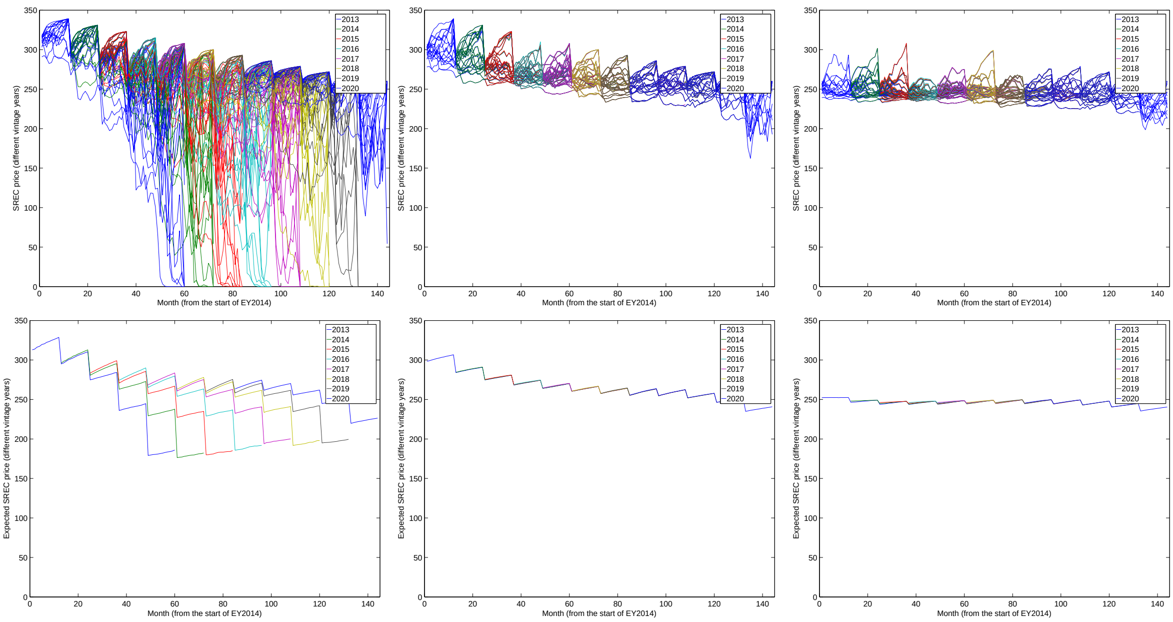


Figure 4: SREC prices in 8 year simulation with $\lambda = 0$, 0.1, and 0.3 from left to right: 20 price sample paths (top row) and average over 10000 simulations (bottom row).

Figure 4 compares SREC prices of different vintage years between a step SACP mechanism and a sloped SACP mechanism. These results are obtained from twenty sample paths, and an average of 10000 simulations with 100 grid points for each state variable $\hat{g}_t$ and b_t (the price feedback parameter a_6 is 7×10^{-4}). To obtain a meaningful comparison in which price volatility can be easily observed, we need prices that are not too high or too low. Therefore, we do not attempt to match our initial conditions to the market, and instead we use $\hat{g}_{13} = R_{14} \exp(-\beta)$ and $\bar{b}_{13} = 0$, which results in mid to high initial prices.

Figure 4 shows that the cliff policy (with $\lambda = 0$) not only produces greater price drops for submitted SRECs (which may then no longer exist), but also far more volatile prices generally. We note that since our initial conditions lead to initial prices fairly near P_t for the cliff policy, a centered tilt under ADAPT tends to shift initial prices downwards towards mid-range levels. Low initial prices would be likely to rise under ADAPT. The bottom row in Figure 4 also shows that there is no price difference between SRECs of different vintages when we used a sloped SACP ($\lambda > 0$), a conclusion that appears to be valid for all positive values of λ. This observation matches our expectation based on results from Section 5. Both of these observations indicate that the sloped mechanism provides a much safer and more attractive environment for investment in solar power.

Also, these figures show that price volatility decreases substantially when λ increases, especially from 0 to 0.1 when a slope is first introduced. In ADAPT, (in most cases) an equilibrium can form between the price of SRECs before and after compliance. This is because if the price after compliance is expected to be higher, a firm would prefer to bank more, accepting a higher penalty today while reducing the price of SRECs in the future, until these two values are balanced. As we have seen, when using higher λs, we effectively increase the total compliance cost paid. However, price levels do not necessarily follow, and in this example they appear to fall. This is logical for the case of tilting a slope when the market is closer to undersupply than oversupply, as is the case here.

Keeping R fixed, increasing λ corresponds to imposing a higher total payment on society, as Theorem 5.11 suggests. However, one could also introduce a slight decrease in R when increasing λ (since the effective full requirement is $(1+\lambda)R$) in order to counteract higher compliance costs, but then also lowering price levels. One might seek a balance between higher payments and less volatility. It seems that the main goal of making solar energy more attractive for investment can be achieved by a market mechanism that produces relatively high and less volatile prices with no price difference between various vintages. Our analysis suggests that an ADAPT mechanism with even a small positive λ can provide these significant benefits, without a major impact on the total cost of compliance. Regulators might also consider a somewhat higher λ to account for possible imperfections in the market's ability to follow the optimal submission policy.

7.2 Adaptive requirements

A challenge faced by policy makers is the design of a fixed target schedule, which appears to require estimating a highly uncertain rate of market adoption many years in advance. Such a strategy is based on a forecast of behaviour, and is not adaptable to many different outcomes, such as the possibility of changes in the cost of installing solar and the behavior of the market. We can circumvent this problem by allowing the requirements to adapt to the current generation level. In our model, we use a combination of a fixed rule and an adaptive rule:

$$R_y = \tilde{R}_y + \alpha \ (b_{y-1} - \tilde{R}_{y-1}), \qquad 0 \le \alpha \le 1, y \in \mathbb{N}.$$

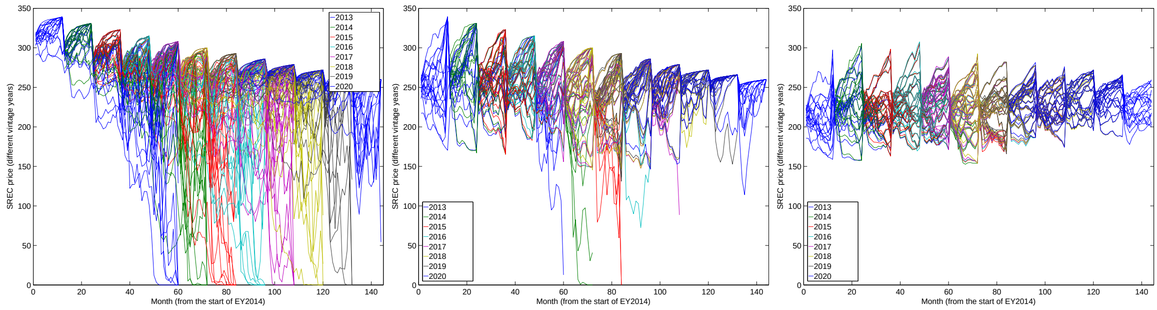


Figure 5: 20 price sample paths for an 8 year simulation with (λ, α) tuples (0,0), (0.3,0), and (0.3,0.5).

According to this mechanism, there are two sets of requirements. The base requirement level $\tilde{R}_y$ is a fixed number determined by the regulators and known to market participants, and the adaptive requirement R_y is the effective requirement for energy year y. According to this rule, a portion of last year's surplus (shortage) is added to (deducted from) the already known base requirement $\tilde{R}_y$. Note that surplus and shortage is defined based on the difference between the total number of available SRECs and the base requirement. One may expect that such an adaptive mechanism can yield less volatile prices by enforcing a balanced growth for generation and requirement levels.

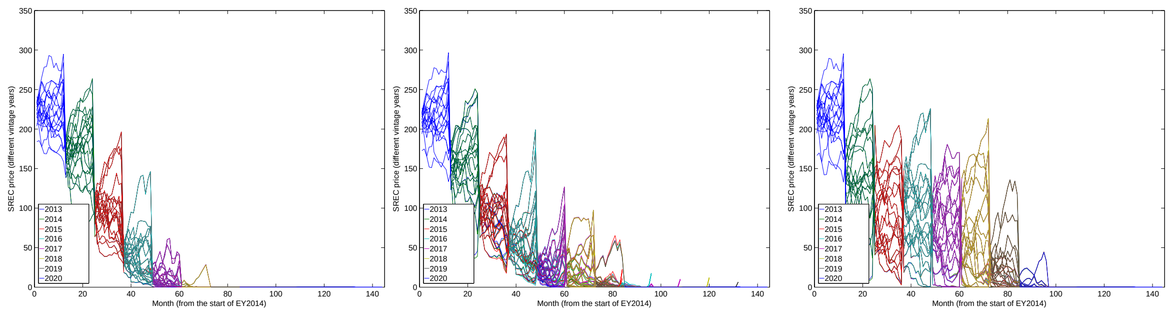


Figure 6: 20 sample paths for original requirements and $\alpha = 0,\ 0.5,\ 1$ respectively ($\lambda = 0.3$, $\hat{g}_{13} = R_{14}\exp(-\beta)$)

As discussed earlier in Section 4, modeling a market design with adaptive requirements requires one extra dimension in our dynamic program. Although this extra dimension increases the computational time significantly (from around one hour to around a week with 50 grid points for each dimension), we still are able to solve this problem using an exact dynamic programming approach. Figure 5 investigates the impact of the adaptive mechanism (for $\alpha = 0.3$), both without a slope ($\lambda = 0$) and in conjunction with the slope ($\lambda = 0.5$). According to these results, an adaptive cliff policy also reduces price volatility and avoids the risk of very low SREC prices in comparison with a simple cliff policy (the current mechanism). Adaptive requirements, however, appear to reduce the price volatility less than a simple sloped policy, while also counteracting some of the effect of the slope when used in conjuction (compare with Figure 4). This final rather surprising result can be attributed to the weakening effect that $\alpha > 0$ has on the ability of a banking decision to lower future expected prices, a mechanism central to the price stabilizing features of ADAPT.

Figure 6 shows SREC prices for 20 sample paths of another experiment on a sloped policy ($\lambda = 0.3$) with and without adaptive requirements ($\alpha = 0$, $\alpha = 0.5$, and $\alpha = 1$). In this experiment the original requirement schedule is used, and thus the rate of generation growth is not in line with the rate of requirement growth. The adaptive requirement mechanism ($\alpha = 0.5$) delays the rate of convergence to zero for a couple of years, but it cannot completely stop it. This can be better explained if we refer to the case with $\alpha = 1$. When $\alpha = 1$, any value of positive surplus would be added to the next year requirement, and this would neutralize the effect of banked SRECs. However, the long-term rate of generation is still more than the requirement and thus prices fall to zero but slower in comparison with the cases of $\alpha = 0$ and 0.5. The price variability increases because adaptivity counteracts the effect of banking, and as we discuss in the next section, lack of banking possibility can increase price volatility.

All in all, although the adaptive requirements can increase price volatility, they can also, to some extent, correct a lack of foresight by policymakers in predicting the rate of generation and ensure more balanced prices in the long term, without the need for frequent regulatory fixes.

7.3 More banking years

Another strategy chosen by the regulators during the lifetime of the NJ SREC market has been increasing the number of banking years. This strategy firstly ensures higher SREC prices as a result of extra years of validity. Secondly, this can stabilize prices from falling too low, by striking a balance among different energy years.

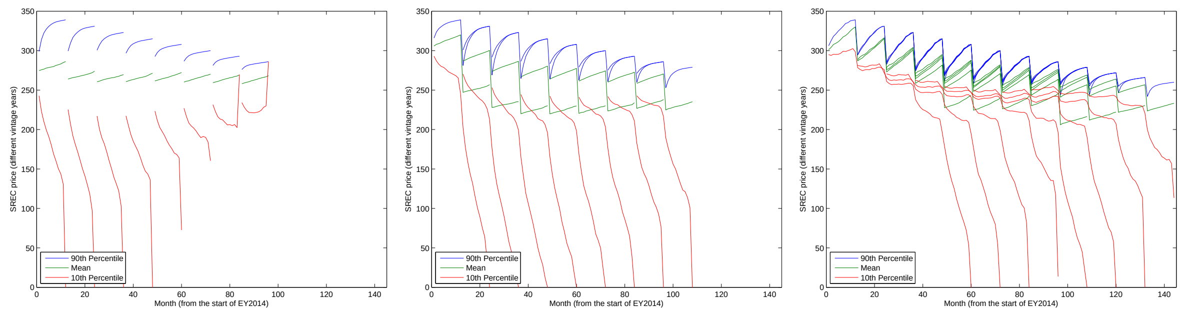


Figure 7: The mean, 10th and 90th percentiles of SREC prices for 10000 simulations with 1, 3, and 5 years of lifetime (from left to right), with $\lambda = 0, \alpha = 0$) and each vintage plotted separately.

Figure 7 shows the results of an experiment on the number of banking years. Note that at each time, different SREC vintages may exist. The same colors of lines are used for each vintage, however, the lines with higher prices correspond to newer vintages. According to this figure, the difference between the 10th and the 90th percentile of SREC prices of the newer vintages, which are the majority of SRECs at each time, decreases with a higher number of banking years. For example, with five years of lifetime, prices of the majority of SRECs (zero or one year old SRECs) are much less volatile than those of one or two years of lifetime.

8 Conclusion

The current SREC market design produces volatile prices and this reduces the attractiveness of this market for investment. This itself creates other problems such as reducing competitiveness of the market and therefore inefficiency in SREC generation. We propose the ADAPT policy which uses a sloped SACP function. We develop a dynamic programming model in order to predict collective market behaviour, and we derive and prove some of the properties of this market such as the collective SREC submission policy. We then use this submission policy and other market properties in a pricing model that enables us to compare the performance of different market designs in terms of price level and volatility. The results of our experiments show that the ADAPT policy (particularly the sloped SACP function) can significantly mitigate the risk of sudden price drops and high volatility, thus ensuring greater market stability and reliability in the long run.

References

Amundsen, E., Baldursson, F., and Mortensen, J. (2006). Price volatility and banking in green certificate markets. *Environmental & Resource Economics*, **35**, 259–287.

Berry, D. (2002). The market for tradable renewable energy credits. *Ecological Economics*, **42**(3), 36917379.

Bird, L. and Reger, A. (2013). Incentivizing distributed solar: Best practices. *NREL*.

Bird, L., Heeter, J., and Kreycik, C. (2011). Solar renewable energy certificate (SREC) markets: Status and trends. *Contract*, **303**, 275173000.

Burns, J. E. and Kang, J.-S. (2012). Comparative economic analysis of supporting policies for residential solar PV in the united states: Solar renewable energy credit (SREC) potential. *Energy Policy*, **44**, 21717225.

Carmona, R., Fehr, F., Hinz, J., and Porchet, A. (2010). Market designs for emissions trading schemes. *SIAM Review*, **52**, 403–452.

Coulon, M., Khazaei, J., and Powell, W. B. (2015). Smart-srec: A stochastic model of the new jersey solar renewable energy certificate market. *Journal of Environmental Economics and Management*, **73**, 13 – 31.

Cramton, P. and Stoft, S. (2005). A capacity market that makes sense. *The Electricity Journal*, **18**(7), 43–54.

Deaton, A. and Laroque, G. (1996). Competitive storage and commodity price dynamics. *Journal of Political Economy*, **104(5)**, 896–923.

DESIRE (2013). New jersey solar renewable energy certificates (SRECs).

Ehrenmann, A. and Smeers, Y. (2011). Generation capacity expansion in a risky environment: A stochastic equilibrium analysis. *Operations Research*, **59**(6), 1332–1346.

Felder, F. A. and Loxley, C. J. (2012). The implications of a vertical demand curve in solar renewable portfolio standards. Center for Research in Regulated Industries, Rutgers University.

Glickstein, J. (2013). New yorks proposed solar renewable energy market: Lessons learned from other states17 SREC markets and recommendations in moving forward. *Pace Environmental Law Review Online Companion*, **3**(1), 26.

Hitzemann, S. and Uhrig-Homburg, M. (2014). Equilibrium price dynamics of emission permits. working paper, available at SSRN 1763182.

Hobbs, B., Hu, M.-C., Inon, J., Stoft, S., and Bhavaraju, M. (2007). A dynamic analysis of a demand curve-based capacity market proposal: The pjm reliability pricing model. *Power Systems, IEEE Transactions on*, **22**(1), 3–14.

Holt, C. A. and Shobe, W. (2016). Price and quantity collars for stabilizing emissions allowance prices: Laboratory experiments on the {EU} {ETS} market stability reserve. *Journal of Environmental Economics and Management*, pages –.

Hustveit, M., Frognera, J. S., and Fleten, S.-E. (2015). Tradable green certificates for renewable support: The role of expectations and uncertainty. working paper.

Jacoby, H. D. and Ellerman, A. D. (2004). The safety valve and climate policy. *Energy Policy*, **32**(4), 481–491.

Kildegaard, A. (2008). Green certificate markets, the risk of over-investment, and the role of long-term contracts. *Energy Policy*, **36**(9), 3413173421.

Kollenberg, S. and Taschini, L. (2015). The european union emissions trading system and the market stability reserve: Optimal dynamic supply adjustment. working paper, available at SSRN 2610213.

Maeda, A. (2004). Impact of banking and forward contracts on tradable permit markets. *Environmental Economics and Policy Studies*, **6**(2), 81–102.

Marchenko, O. V. (2008). Modeling of a green certificate market. *Renewable Energy*, **33**(8), 1953171958.

Naroditsky, J. (2013). Response to new yorks proposed solar renewable energy market: Lessons learned from other states17 SREC markets and recommendations in moving forward. *Pace Environmental Law Review*, **30**(3), 1276.

NJCEP (2013). SREC pricing | NJ OCE web site.

Pirrong, C. (2014). *Commodity Price Dynamics A Structural Approach*. Cambridge University Press.

Scheinkman, J. A. and Schechtman, J. (1983). A simple competitive model with production and storage. *The Review of Economic Studies*, **50**(3), 427–441.

Stoddard, R. and Adamson, S. (2009). Comparing capacity market and payment designs for ensuring supply adequacy. In *System Sciences, 2009. HICSS '09. 42nd Hawaii International Conference on*, pages 1–8.

Wiser, R., Barbose, G., and Holt, E. (2011). Supporting solar power in renewables portfolio standards: Experience from the united states. *Energy Policy*, **39**(7), 3894173905.

Witmer, L., Idrisu, B. D., Bayrakci, M., Brownson, J. R., and Blumsack, S. (2012). The actual value of solar electricity (photovoltaics) in urban settings: Real-time pricing, srecs, and tax credits.

Xu, L., Deng, S.-J., and Thomas, V. M. (2014). Carbon emission permit price volatility reduction through financial options. *Energy Economics*, pages –.