Demand Management

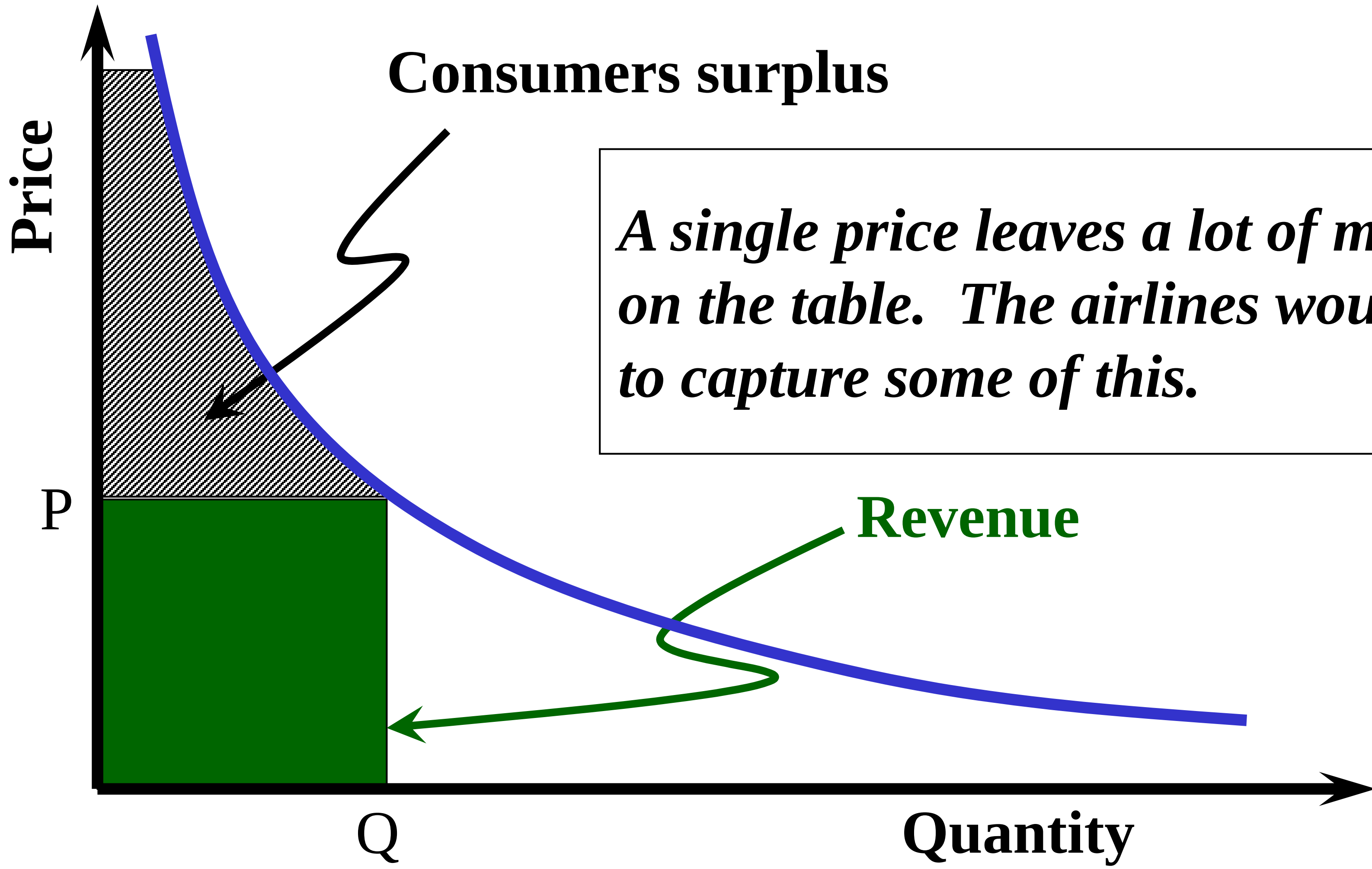
- Principles of demand management
- Airline yield management
- Determining the booking limits
  - » A simple problem
  - » Stochastic gradients for general problems

Principles of demand management

- Issues:
  - » Customers are not all the same
  - » The highest paying customers will not fill up available capacity.
  - » The lowest paying customers will not cover the full costs of operations.
  - » Marginal costs may be much lower than average costs.

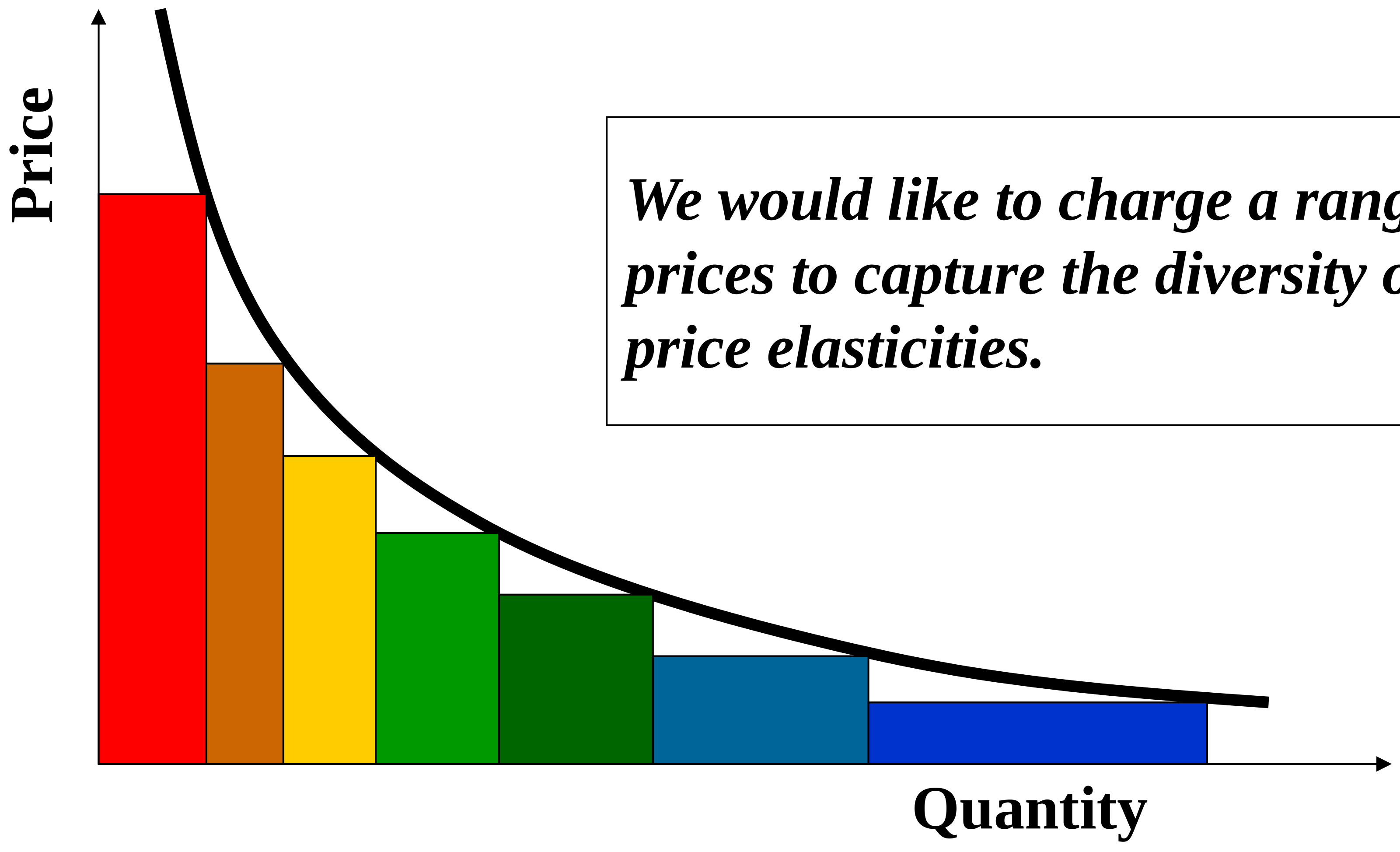
Principles of demand management

- From microeconomics:



A single price leaves a lot of money on the table. The airlines would like to capture some of this.

Principles of demand management



We would like to charge a range of prices to capture the diversity of price elasticities.

Principles of demand management

- Modes of control:
  - » Primal
    - Limit access to the system (now)
    - Reservations (future)
  - » Dual
    - Pricing
    - Service (waiting in line)
  - » Informational
    - Advertising
    - Promotions
  - » Combinations:
    - Coupons - provide discounts (dual) with restricted services (primal) to customers who hold coupons (informational).

Principles of demand management

- Market differentiating characteristics used in practice:
  - » Advance notice (pre-booking, reservations)
  - » Length of stay, stay over Saturday, etc.
  - » Willingness to accept nonrefundable tickets
  - » Service
    - Additional amenities (first class, business class)
    - Pre-board privileges (frequent flier status)

Principles of demand management

- Applications of demand management
  - Airlines
  - Hotels
  - Universities (with rolling admissions)

Lecture outline - Demand Management

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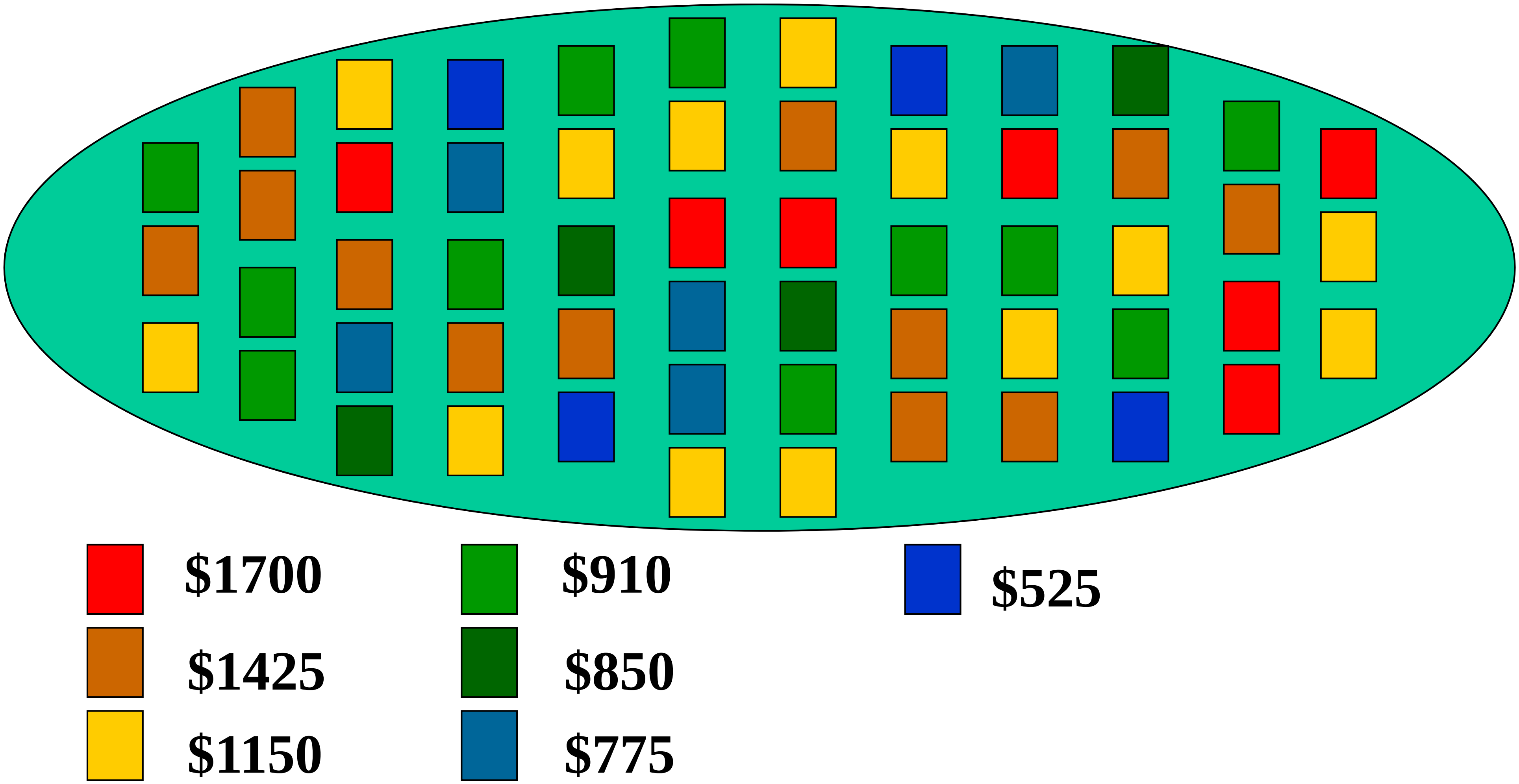
Airline yield management

- Overview
  - » Old system:
    - Airlines establish a fare (or two, such as coach and first class)
    - Sell seats at the fare until all seats are sold or the flight departs.
    - Because of no-shows, airlines may sell too many seats to increase load factors.
  - » Issue:
    - Business travelers are willing to pay much more, but want the flexibility of making plans at the last minute.
    - Different customers want different "products" measured in terms of price and service.
  - » Solution:
    - Offer blocks of seats at increasingly higher prices. The lower demand for higher priced seats has the effect of making seats available for last-minute business travelers.

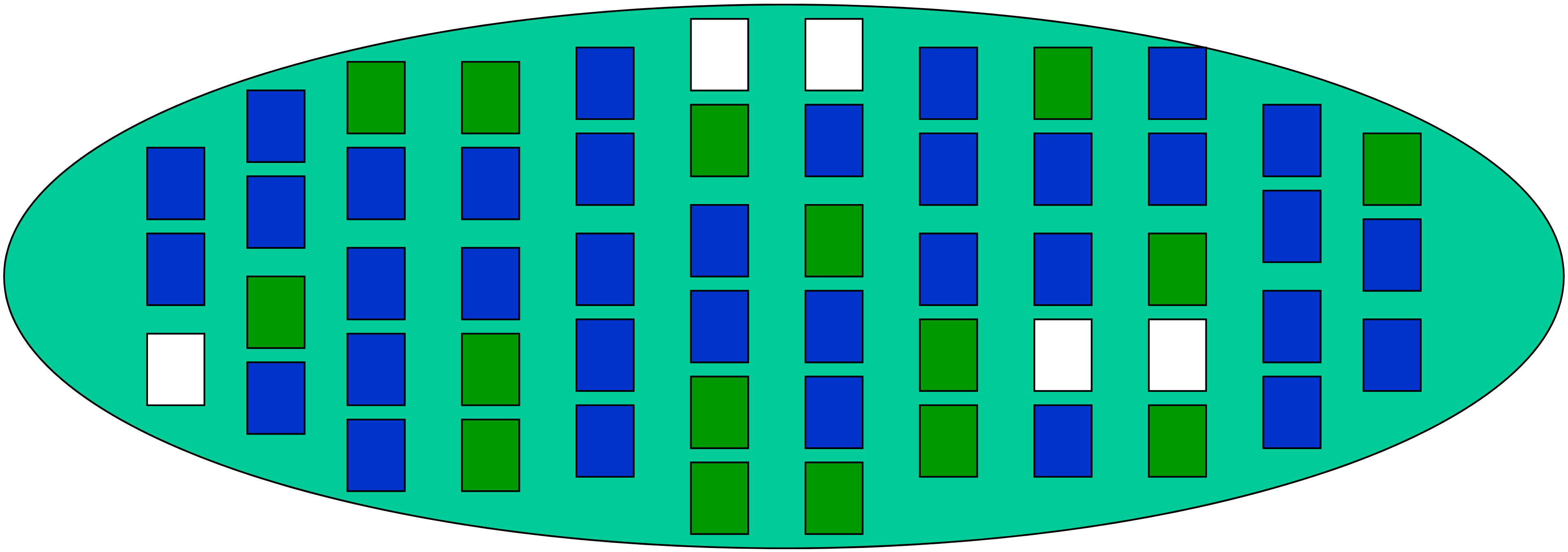
Airline yield management

- Booking limits
  - » Blocks of seats are offered at different prices:



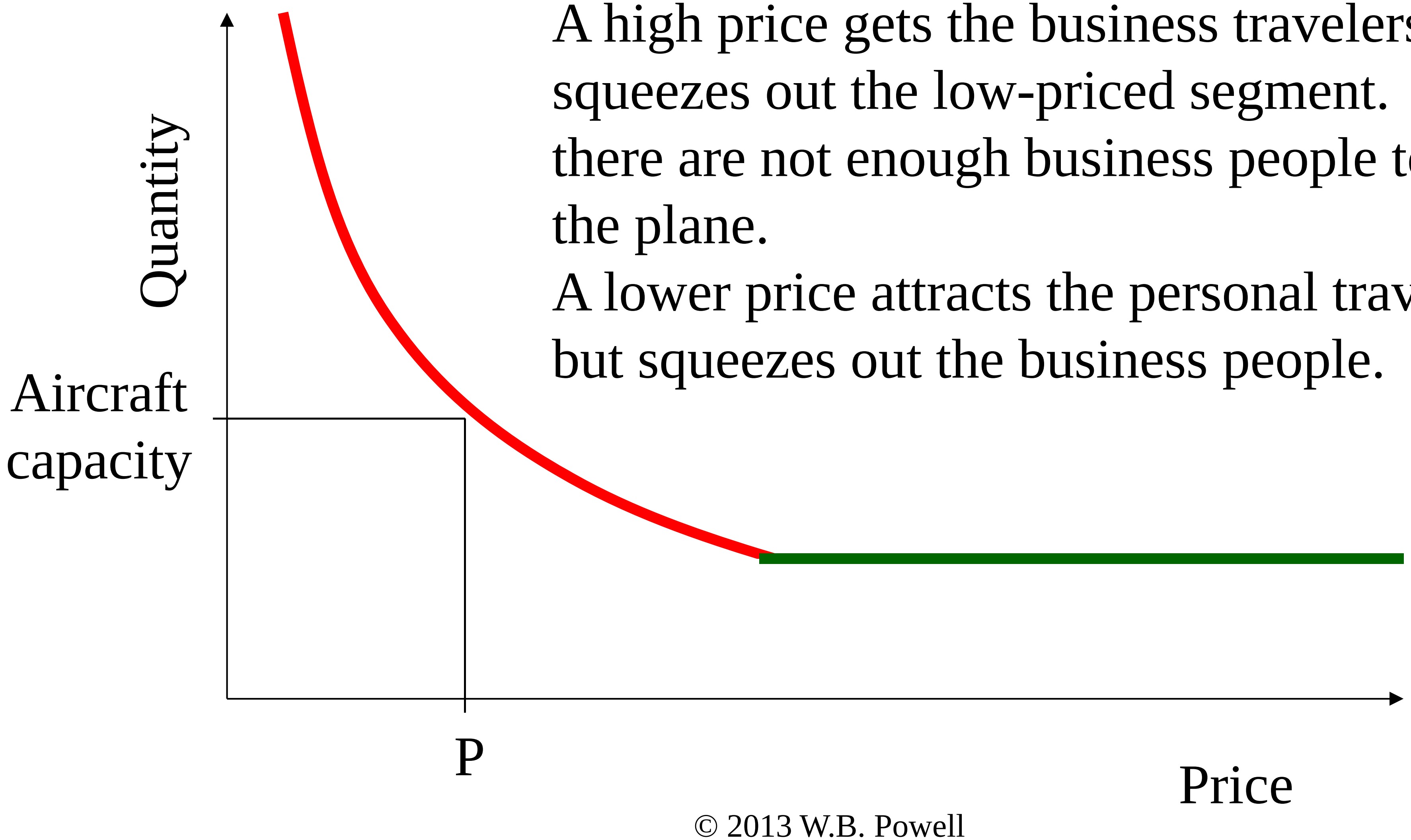
Airline yield management

- Consider a two-class problem:



Airline yield management

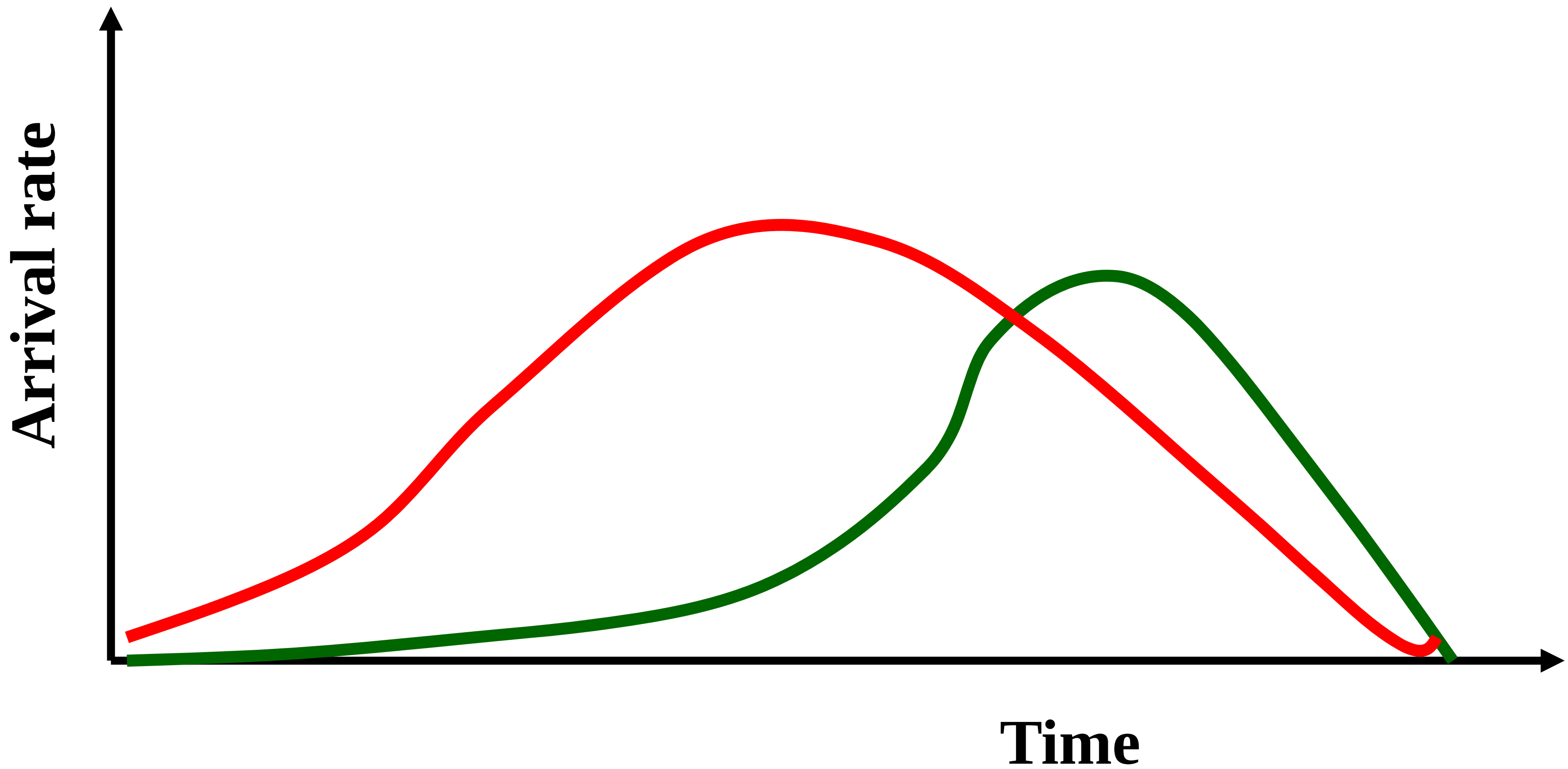
- What price to charge?



A high price gets the business travelers, but squeezes out the low-priced segment. But there are not enough business people to fill the plane.
A lower price attracts the personal travelers, but squeezes out the business people.

Airline yield management

- Different customer classes vary in terms of their booking process.



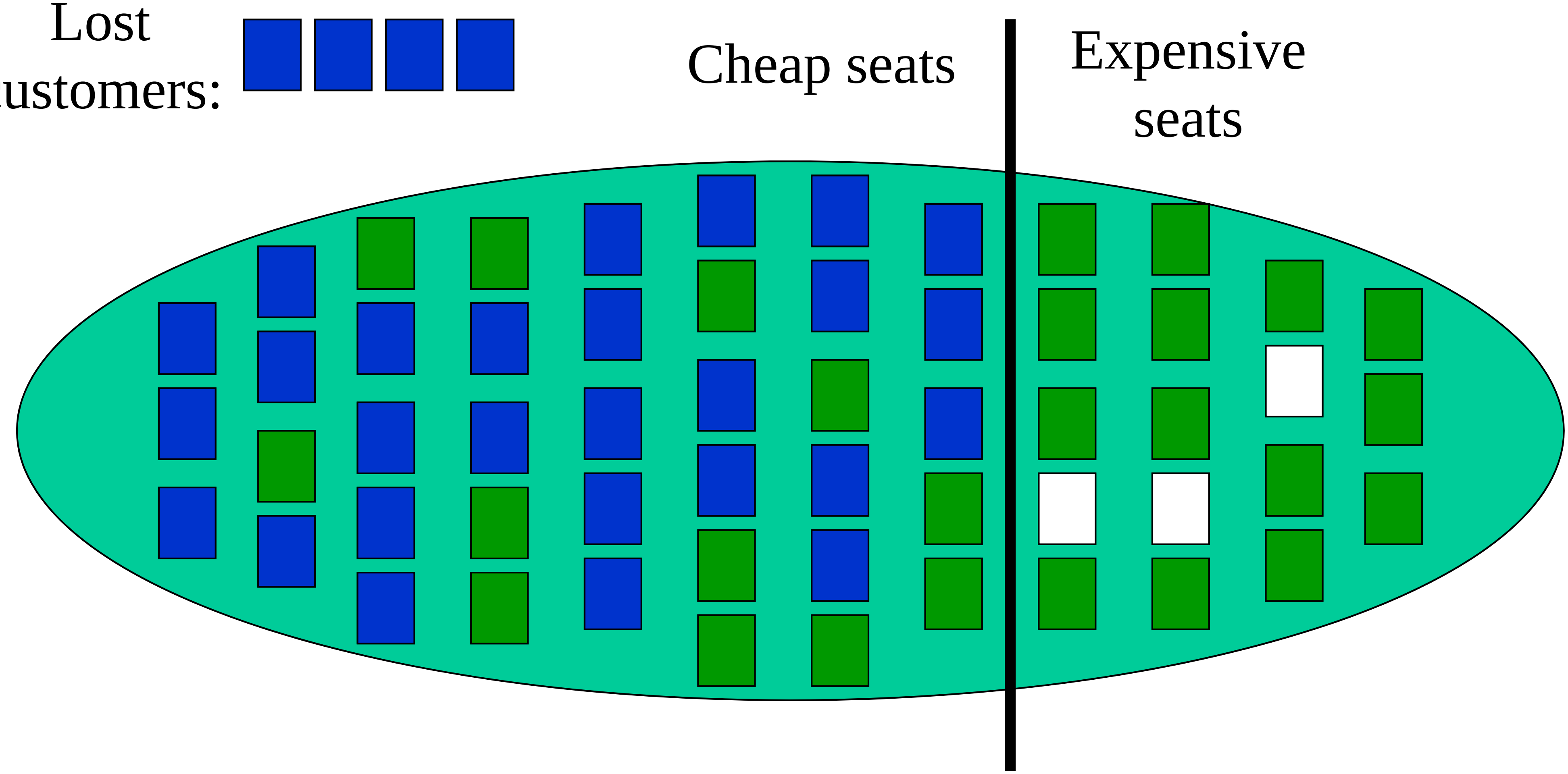
Airline yield management

- Demand management techniques
  - Informational
    - Advertising services
    - Discounts
  - Dual (pricing)
    - Setting the price for seats
    - Pricing differentiated services
  - Primal (booking limits)
    - We have to limit the number of seats we sell at each fare.

Airline yield management

- Consider a two-class problem:

Lost customers:



How do we decide if we picked the right booking limit?

Airline yield management

- Challenge question:
  - » What happens when the allotment of “cheap seats” is set too high? Too low?

Airline yield management

- Notation:

Activity variables:

$J \quad =$ Set of fare classes. j = 1 is the "highest" fare class.

$D_j(\omega) =$ (random) Demand for fare class j

$f_j(y) =$ p.d.f. of demand for fare class j.

Decision variables:

$U_j \quad =$ Limit (upper bound) on number of reservations allowed for fare class j.

$x_j(\omega) =$ Number of reservations made for fare class j.

Parameters:

$C \quad =$ Capacity of the aircraft

$p_j \quad =$ Fare (price) for fare class j $(p_j \geq p_{j+1})$

Airline yield management

- Objective:

We would like to find the booking limits that solve:

$$\max_{U_1 \ldots U_J} E\left\{\sum_{j \in J} p_j x_j(\omega)\right\}$$

subject to limits on how much we can book.

We can think of ω as a single flight, and the expectation is a summation over many flights. We cannot maximize the profits for a single flight, but we would like to maximize profits over many flights.

Airline yield management

- Booking limits:
  - » Separable booking limits- Booking limit j limits reservations for fare class j alone:

$$x_j \le U_j$$

$$\sum_{j \in J} U_j = C = \text{aircraft capacity}$$

  - » Nested booking limits - Booking limit j limits fare class j and all lower fare classes.

$$\sum_{m \ge j} x_m \le U_j$$

$$U_j \ge U_{j+1}$$

$$U_1 = C$$

Airline yield management

A heuristic derivation for booking limits:

Start with a separable (non-nested) policy:

Let:

$$P_j(x) = \text{Prob}[D_j \le x]$$

$$= \int_{y=0}^{x} f_j(y)dy$$

This means that if we set a booking limit at U_j, the probability that we could have exceeded it is $\left(1 - P_j(U_j)\right)$. Now let:

$$\text{EMSR}_j(\text{U}_j) = \text{ The expected marginal seat revenue for fare class j}$$

$$= \text{ The value an additional seat allocated for fare class j.}$$

$$= p_j\left(1 - P_j(U_j)\right).$$

Airline yield management

We start by expressing the objective function :

$$F(U) = \max_{U_j} E\{F(U,\omega)\}$$

subject to : $\sum_{j \in J} U_j = C$

where :

$$F(U,\omega) = \sum_j p_j \min\{D_j(\omega), U_j\}$$

Now consider the derivative :

$$\frac{\partial F(U,\omega)}{\partial U_j} = \begin{cases} p_j & D_j(\omega) > U_j \\ 0 & \text{Otherwise} \end{cases}$$

This means that :

$$E\left\{\frac{\partial F(U,\omega)}{\partial U_j}\right\} = p_j \text{Prob}\left[D_j(\omega) > U_j\right] = p_j\left(1 - P_j(U_j)\right)$$

Airline yield management

We can formulate the problem as an unconstrained problem by relaxing the constraint:

$$F^L(U,\lambda) = \max_{U_j} E\left\{F(U,\omega)\right\} + \lambda\left(C - \sum_{j\in J} U_j\right)$$

At optimality, we expect to find $\frac{\partial F^L(U,\lambda)}{\partial U_j} = 0$. This means that:

$$p_j\left(1\text{-}P_j(U_j)\right) - \lambda = 0 \quad \text{for each j.}$$

Or:

$$p_j\left(1\text{-}P_j(U_j)\right) = \lambda$$

What is the economic interpretation of this equation?

Airline yield management

- The EMSR algorithm:

For a given value of λ, find the booking limit U_j :

$$P_j(U_j) = 1 - \frac{\lambda}{p_j}$$

$$= \frac{p_j - \lambda}{p_j}$$

Given λ, we can find $U_j(\lambda)$ so that this is satisfied.

Now we have to pick λ so that

$$\sum_j U_j(\lambda) = C$$

Since λ is a scalar, just have to try different values.

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Stochastic gradient algorithms

- Nested booking limits:
  - » A sample booking process:

Fare Class	Demand $D(\omega)$	Booking limit	Total reservations	Fare class reservations
5	42	15	15	15
4	38	40	40	25
3	27	80	67	27
2	59	120	120	53
1	19	150	139	19

Stochastic gradient algorithms

Nested booking limits:

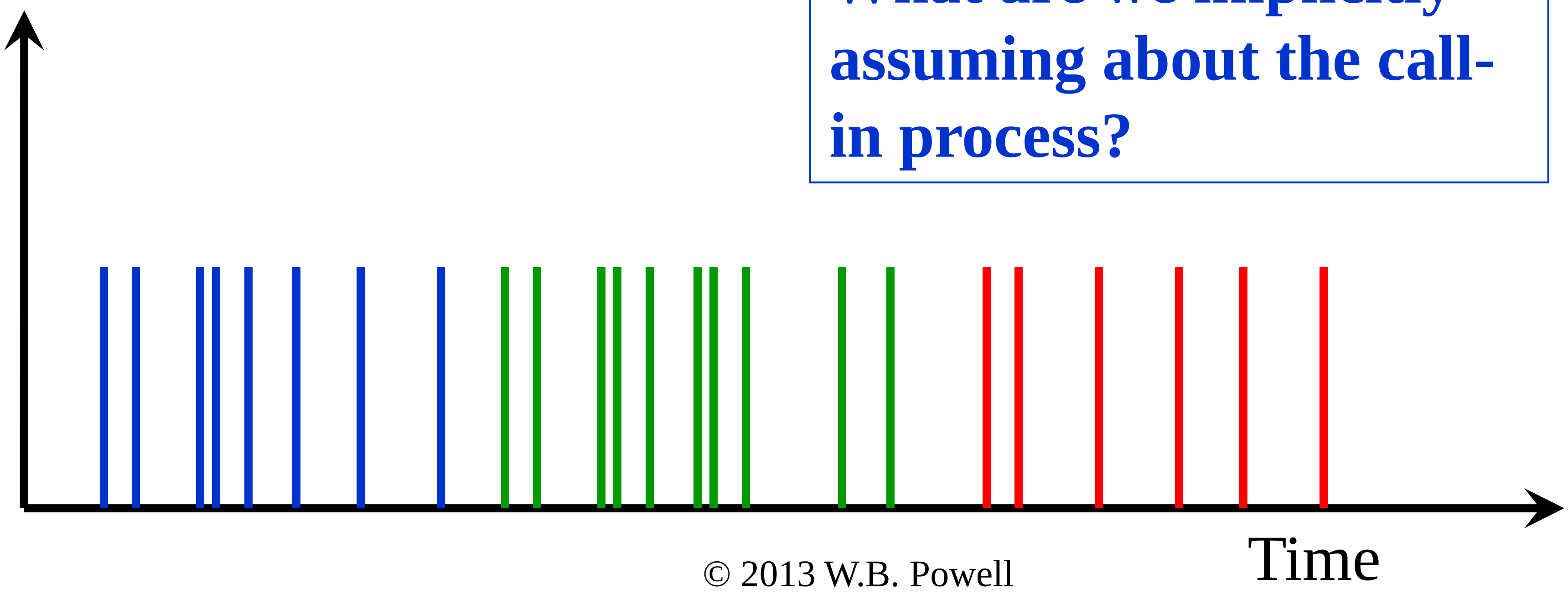
Start by defining the cumulative demand:

$$\hat{D}_j(U,\omega) = \text{ Total reservations made of customer classes j and "lower."}$$

$$= \min\left\{D_j + \hat{D}_{j+1}(\omega), U_j\right\}$$

Note: $\hat{D}_j(U,\omega) - \hat{D}_{j+1}(U,\omega)$ is the number of fare class j reservations that have been accepted.

What are we implicitly assuming about the call-in process?



Stochastic gradient algorithms

For this problem, we again want to solve:

$$F(U) = \max_{U_j} E\left\{F(U,\omega)\right\}$$

where:

$$F(U,\omega) = \sum_{j \in J} p_j \left(\hat{D}_j(U,\omega) - \hat{D}_{j+1}(U,\omega)\right)$$

But how do we find the gradient?

Stochastic gradient algorithms

Consider a two class system...

We start with $U_1 = C$.

$$F(U,\omega) = p_1\left(\hat{D}_1(U,\omega) - \hat{D}_2(U,\omega)\right) + p_2\left(\hat{D}_2(U,\omega)\right)$$

$$= p_1\left(\min\left\{D_1 + \hat{D}_2, U_1\right\} - \hat{D}_2(U,\omega)\right) + p_2\hat{D}_2(U,\omega)$$

Let's now assume that with our low "vacationeers" fare p_2,

$D_2 >> U_2$. In this case, we have:

$$F(U,\omega) = p_1\left(\min\left\{D_1 + U_2, U_1\right\} - U_2\right) + p_2 U_2$$

$$= p_1\left(\min\left\{D_1 + U_2 - U_2, U_1 - U_2\right\}\right) + p_2 U_2$$

$$= p_1 \min\left\{D_1, U_1 - U_2\right\} + p_2 U_2$$

Stochastic gradient algorithms

Taking the derivative with respect to U_2 gives:

$$\frac{\partial F(U,\omega)}{\partial U_2} = -p_1 I_{\{D_1 \geq (U_1 - U_2)\}} + p_2$$

(Remember: $\mathrm{I}_{\{\mathrm{X}\}} = 1$ if event X is true, and 0 otherwise. $\mathrm{E}\left(\mathrm{I}_{\{\mathrm{X}\}}\right)$ is then the probability that X is true.)

Taking expectations gives:

$$E\left[\frac{\partial F(U,\omega)}{\partial U_2}\right] = -p_1 \mathrm{Prob}[D_1 \geq U_1 - U_2] + p_2$$

Recall that $U_1 = C$. Setting the derivative equal to zero, we get:

$$p_1 \mathrm{Prob}[D_1 \geq C - U_2] = p_2$$

Stochastic gradient algorithms

When the problem gets more complicated, we do not get such neat results. Instead, we can resort to our stochastic gradient algorithms.

$$\max E\left\{F(U,\omega)\right\} = \max E\left\{\sum_{j\in J} p_j\left(\hat{D}_j(U,\omega) - \hat{D}_{j+1}(U,\omega)\right)\right\}$$

Assume we have an initial vector U^0. We can find the best value of U using the stochastic gradient iteration:

$$U^{k+1} = U^k + \alpha^k \nabla F(U^k,\omega^k)$$

Stochastic gradient algorithms

- Consider the change in a booking limit:
  - » Assume we change from $U_4=40$ to $U_4=41$.

Fare Class	Demand D(ω)	Booking limit	Total reservations	Fare class reservations	Change
5	42	15	15	15	
4	38	40(41)	40(41)	25(26)	+1
3	44	80(80)	80(80)	40(39)	-1
2	55	120(120)	120(120)	40(40)	
1	27	150(150)	147(147)	27(27)	

Change in profits is p_4-p_3.

What if U_3 was not binding?

Stochastic gradient algorithms

- Consider the change in a booking limit:
  - » What if U_3 is not binding, but U_2 is?

Fare Class	Demand D(ω)	Booking limit	Total reservations	Fare class reservations	Change
5	42	15	15	15	
4	38	40(41)	40(41)	25(26)	+1
3	34	80(80)	74(75)	34(34)	
2	55	120(120)	120(120)	46(45)	-1
1	27	150(150)	147(147)	27(27)	

Now the change in profits is p_4-p_2.

Stochastic gradient algorithms

- Consider the change in a booking limit:
  - » What if nothing else is binding?

Fare Class	Demand D(ω)	Booking limit	Total reservations	Fare class reservations	Change
5	42	15	15	15	
4	38	40(41)	40(41)	25(26)	+1
3	34	80(80)	74(75)	34(34)	
2	40	120(120)	114(115)	40(40)	
1	27	150(150)	141(142)	27(27)	

Now the change in profits is p_4.

Stochastic gradient algorithms

- Consider the change in a booking limit:
  - » What if U_4 is not binding, but U_2 is?

Fare Class	Demand D(ω)	Booking limit	Total reservations	Fare class reservations	Change
5	42	15	15	15	
4	20	40(41)	35(35)	20(20)	
3	34	80(80)	69(69)	34(34)	
2	55	120(120)	120(120)	51(51)	
1	27	150(150)	147(147)	27(27)	

Now there is no change in profits.

Stochastic gradient algorithms

So we have a formula for the gradient when increasing U_i:

Let:

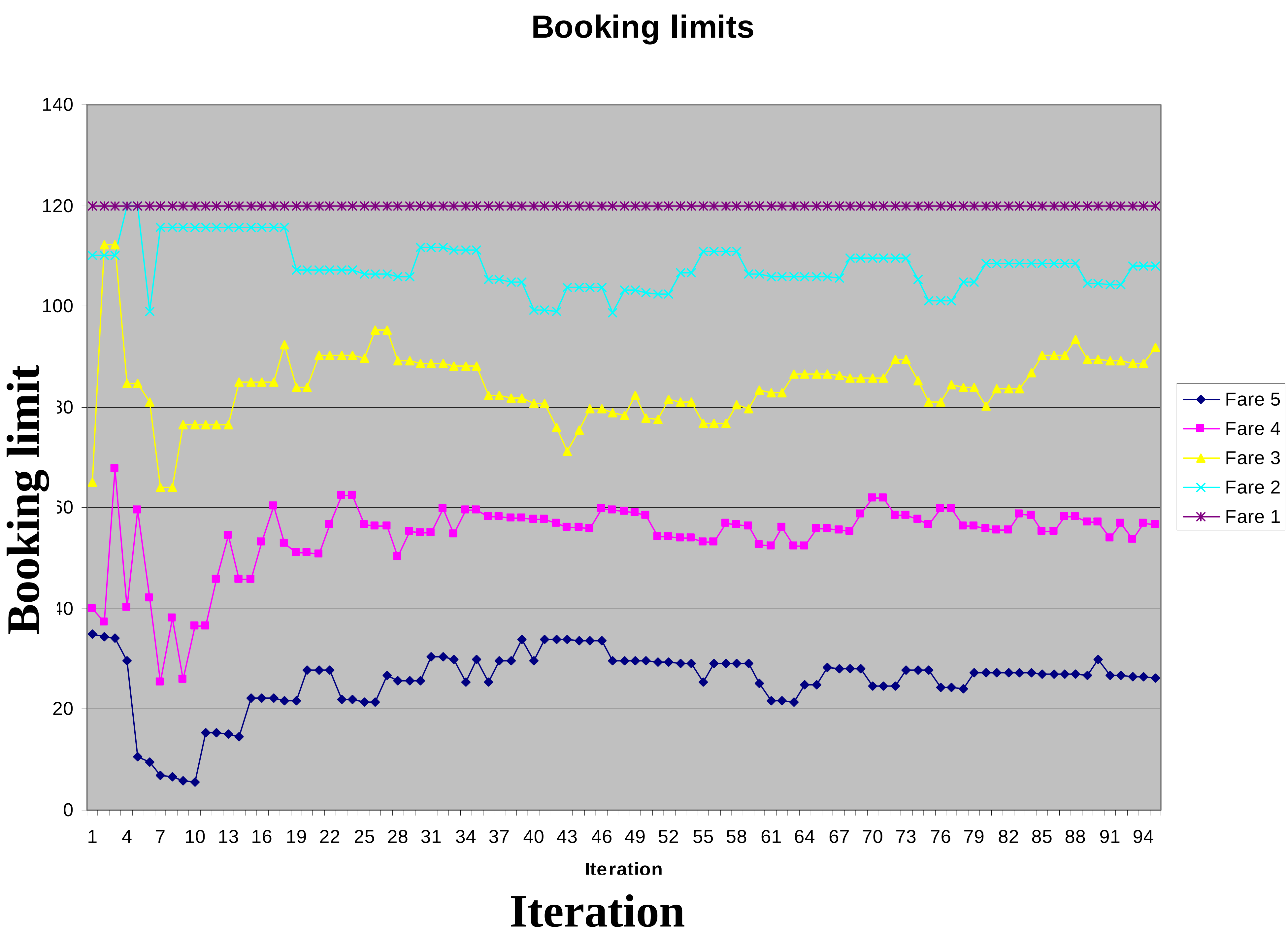
$$X^+(\omega) = \begin{cases} p_i & \text{If booking limit } U_i \text{ is binding.} \\ 0 & \text{Otherwise.} \end{cases}$$

$$X^-(\omega) = \begin{cases} p_j & \text{If } U_i \text{ is binding, and } U_j \text{ is binding, and } j < i \text{ is the first class} \\ & \text{whose booking limit is binding.} \\ 0 & \text{If no other booking limit for a higher fare class is binding.} \end{cases}$$

We can now express the gradient using:

$$\frac{\partial F(U,\omega)}{\partial U_i} = X^+(\omega) - X^-(\omega).$$

Since X^+ and X^- depend on the sample realization of demands, they are random variables.


Booking limits
Booking limit
140
120
100
0
20
Fare 5
Fare 4
Fare 3
Fare 2
Fare 1
1
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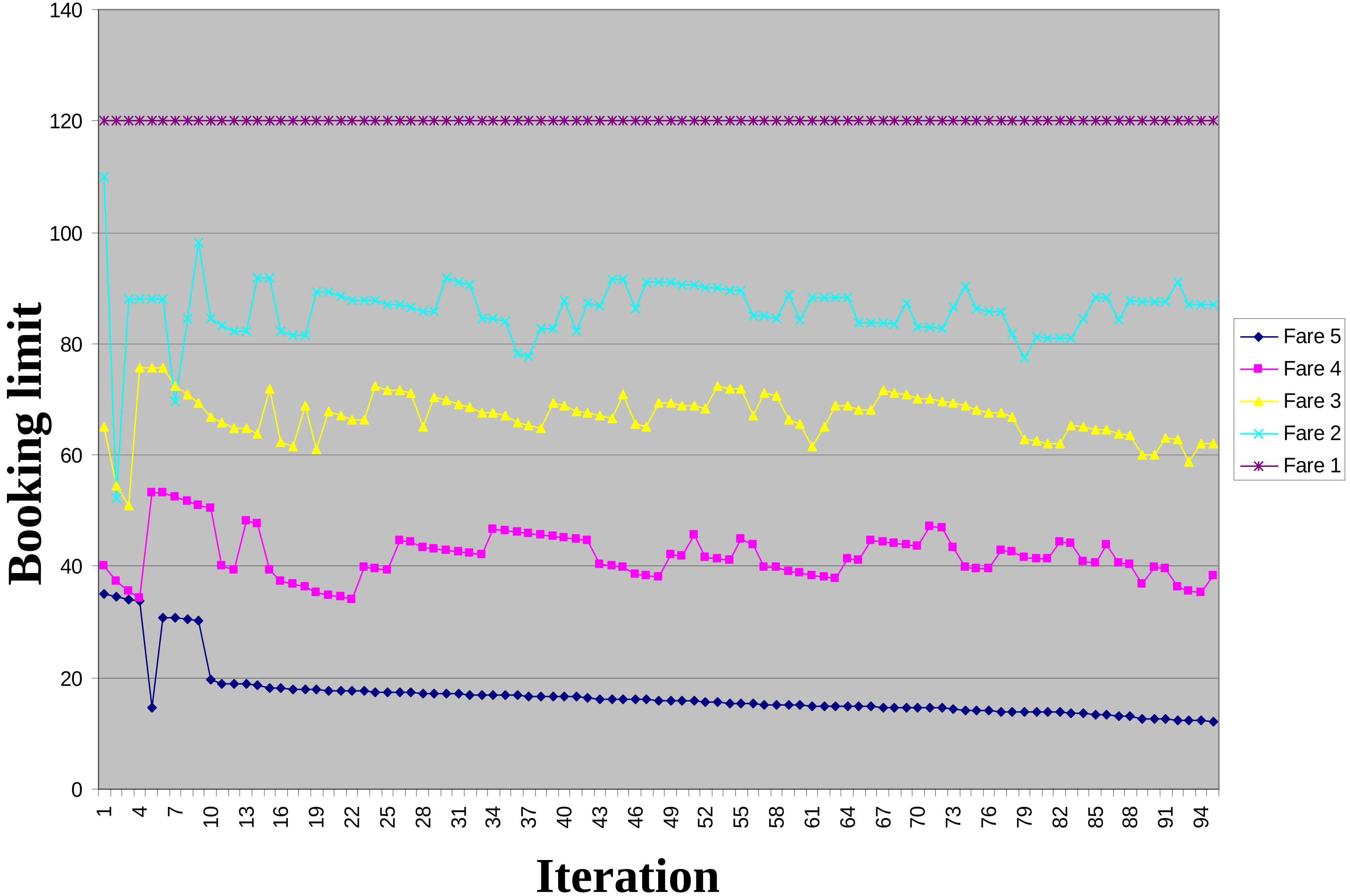
Iteration
Iteration

Higher demand for fare class 1



Lecture outline - Demand Management

- Issues in demand management

Extensions

- The no-show problem
  - » How can we manage this?
- The fairness issue
  - » Paying different amounts for the “same” seat
  - » In what way are seats different?
- Managing complexity
  - » 1000 markets
  - » 30,000 fares!