Energy Storage Benchmark Problems

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1 Purpose

In this document, we lay out the model for a general energy storage control problem and then design a set of benchmark problems which can be solved exactly. The exact solution to each one of these problems is included in the accompanying .zip file. This is meant to be a collection of problems that anyone can use to asses the optimality of their algorithm.

We understand that real energy markets are much more complex than in this setting but we are confident that the material presented in this document serves as a generally realistic benchmark for energy storage control algorithms. Furthermore, we want to note that some of the model parameters were picked fairly arbitrarily, though we made sure they led to interesting models in the context of energy storage. Ultimately, the purpose of this document is to serve as a algorithmic benchmark and not an exact model of wind energy storage.

Furthermore, variables are presented as unitless with the understanding that the appropriate units of energy, power, price, etc. are implied. For example, one can think of time as hr, energy as MWh, power as MW, price as $, etc. or any other unit system as long as all variables are of consistent dimensions.

2 A General Storage Problem

The network consists of a single energy storage device which is connected to a wind farm and to the electricity grid. Electricity may flow directly from the wind farm to the storage device or it may be used to satisfy the demand. Energy from storage may be sold to the grid at any given time, and electricity from the grid may be bought to replenish the energy in storage or to satisfy the demand. We let $\mathcal{T} = \{0, \Delta t, 2\Delta t, \ldots, T - \Delta t, T\}$ be a finite time horizon.

2.1 The State of The System

The variable $S_t = (R_t, E_t, D_t, P_t)$ describes the state of the system at time t is given by:

- R_t : The amount of energy in the storage device at time t.
- E_t : The net amount of wind energy available at time t.
- D_t : The aggregate energy demand at time t.
- P_t : The price of electricity at time t in the spot market.

2.2 The Decisions

At any point in time, the decision is given by the column vector $x_t = (x_t^{WD}, x_t^{RD}, x_t^{GD}, x_t^{WR}, x_t^{GR}, x_t^{RG})$, where x_t^{IJ} is the amount of energy transferred from I to J at time t. The superscript W stands for wind, D for demand, R for storage and G for grid.

2.3 The Constraints

We require that all components of x_t be nonnegative for all t. We let R^{max} be the total capacity of the battery, η^c and η^d be the charging and discharging efficiencies, respectively, and γ^c and γ^d be the maximum charging and discharging rates, respectively. For energy storage, at any time t we also require that the total amount of energy stored in the device from the wind does not exceed the capacity available,

$$x_t^{WR} + x_t^{GR} \leq R^{max} - R_t. \tag{1}$$

We also make the assumption that all demand at t must be satisfied at time t:

$$x_t^{WD} + \eta^d x_t^{RD} + x_t^{GD} = D_t. \tag{2}$$

Additionally, the amount withdrawn from the device at time t to satisfy demand plus any amount of energy sold to the grid after satisfying demand must not exceed the amount of energy that is available in the device when we make the decision to store or withdraw:

$$x_t^{RD} + x_t^{RG} \leq R_t. \tag{3}$$

The total amount of energy charged to or withdrawn from the device is also constrained by the maximum charging and discharging rates:

$$x_t^{WR} + x_t^{GR} \leq \gamma^c, \tag{4}$$

$$x_t^{RD} + x_t^{RG} \leq \gamma^d. \tag{5}$$

Finally, flow conservation requires that:

$$x_t^{WR} + x_t^{WD} \leq E_t. \tag{6}$$

The feasible action space, $\mathcal{X}_t$, is the convex set defined by (1)-(6). We let $X_t^\pi(S_t)$ be the decision function that returns $x_t \in \mathcal{X}_t$, where $\pi \in \Pi$ represents the type of policy (which we determine later).

2.4 The Exogenous Information Process

For the purpose of this model, $W_t = (\hat{E}_t, \hat{P}_t)$. Note that the demand is assumed to be fixed (though not necessarily constant).

$\hat{E}_t =$ The change in the energy between times $t - \Delta t$ and t.

$\hat{P}_t =$ The change in the price of electricity between times $t - \Delta t$ and t.

To avoid violating the nonanticipativity condition, we assume that any variable that is indexed by t is $\mathcal{F}_t$-measurable. As a result, W_t is defined to be the information that becomes available between times $t - \Delta t$ and t. A sample realization of W_t is denoted $W_t^n = W_t(\omega^n)$ for sample path $\omega^n \in \Omega$.

2.4.1 The Discrete Uniform Distribution

We let $\mathcal{U}(a, b)$ for $a, b \in \mathbb{R}$ be the uniform distribution which defines the evolution of a discrete random variable X with meshsize ΔX. Then each element in $\mathcal{X} = \{a, a+\Delta X, a+2\Delta X, \ldots, b-\Delta X, b\}$ has the same probability of occurring. The probability mass function is given by:

$$u_X(x) = \frac{\Delta X}{b - a + \Delta X},$$

for all $x \in \mathcal{X}$.

2.4.2 The Discrete Pseudonormal Distribution

Let X be a normally distributed random variable and let $f_X(x; \mu_X, \sigma_X^2)$ be the normal probability density function with mean μ_X and variance σ_X^2. We define a discrete pseudonormal probability mass function for a discrete random variable $\bar{X}$ with support $\mathcal{X} = \{a, a + \Delta X, a + 2\Delta X, \ldots, b - \Delta X, b\}$ as follows, where $a, b \in \mathbb{R}$ are given and ΔX is the mesh size. For $x_i \in \mathcal{X}$ we let:

$$g_{\bar{X}}(x_i; \mu, \sigma^2) = \frac{f_X(x_i; \mu_X, \sigma_X^2)}{\sum_{x_j=0}^{|\mathcal{X}|} f_X(x_j; \mu_X, \sigma_X^2)}$$

be the probability mass function corresponding to the discrete pseudonormal distribution. We say that $\bar{X} \sim \mathcal{N}(\mu_X, \sigma_X^2)$ if $\bar{X}$ is distributed according to the discrete pseudonormal distribution. We recognize this is non-standard notation but it simplifies the notation in this document.

We include sample m-files $createPriceProbability.m$ and $createWindProbability.m$ for creating the pseudonormal probability density function and the cumulative distribution function.

2.5 The Transition Function

The transition function is given by $S_{t+\Delta t} = S^M(S_t, x_t, W_{t+\Delta t})$. The transition function for the energy in storage is given by:

$$R_{t+\Delta t} = R_t + \phi^T x_t,$$

where $\phi = (0, -1, 0, \eta^c, \eta^c, -1)$ is an incidence column vector that models the flow of energy from one node to another. The transition dynamics for the wind, price and demand processes are given in sections 4.1-4.3.

2.6 The Objective Function

The function $C(S_t, x_t)$ represents the contribution from being in the state S_t and making the decision x_t at time t. Assuming that the demand at time t must always be satisfied at time t, our contribution at time t

is just the total amount of money paid or collected when we transfer energy to and from the grid at time t:

$$C(S_t, x_t) = P_t D_t - P_t(x_t^{GR} - \eta^d x_t^{RG} + x_t^{GD}) - c^h(R_t + \phi^T x_t),$$

where $c^h = 0.001$ is a holding cost.

We consider the control problem of maximizing the total un-discounted expected contributions over the finite time horizon $\mathcal{T}$: The objective function is then given by:

$$F^{\pi^*} = \max_{\pi \in \Pi} \mathbb{E}\Big[\sum_{t \in \mathcal{T}} C\big(S_t, X_t^\pi(S_t)\big)\Big]. \tag{7}$$

3 Linear Programming

If the state variable evolves deterministically and the dynamics are known *a priori*, we can solve the control problem using a standard batch linear program (LP):

$$F^* = \max_{x_0, \cdots, x_T} \sum_{t \in \mathcal{T}} C(S_t, x_t),$$

such that $x_t \in \mathcal{X}_t$ for each t and subject to transition dynamics expressed as a set of constraints linking all time points. This formulation is most useful when we can make exact predictions about the wind, demand and price trends. This is hardly ever the case with physical processes that are intrinsically stochastic, but deterministic problems are useful to test the ability of the algorithm to learn the solution in the presence of holding costs, when energy should be stored in the device as latest as possible in order to avoid incurring extra costs. These test problems also allow us to test the capability of the algorithm to learn to store energy in cases where the impact of storing is not felt until hundreds of time periods into the future.

We test different types of deterministic transitions dynamics, as specified in table 1. The actual values for each of these is given in the accompanying dataset, as explained in section 5.

4 Dynamic Programming

In order to solve the stochastic problem exactly, we assume the device is perfectly efficient. For this reason, we use a modified cost function in our simulations:

$$C(S_t, x_t) = P_t D_t - P_t(x_t^{GR} - \rho x_t^{RG} + x_t^{GD}) - c^h(R_t + k \cdot x_t),$$

where $\rho = 0.98$ is a soft cost added to avoid degenerate solutions.

4.1 The Wind Process

The wind process E_t is modeled using a first-order Markov chain:

$$E_{t+\Delta t} = E_t + \hat{E}_{t+\Delta t} \ \forall t \in \mathcal{T} \setminus \{T\},$$

such that $E^{min} \leq E_t \leq E^{max}$, and where $\hat{E}_t$ is either pseudonormally or uniformly distributed (see Table 2). In the case where $\hat{E}_t \sim \mathcal{N}(\mu_E, \sigma_E^2)$, its support is $\{-3, -3+\Delta E, -3+2\Delta E, \ldots, 0, \ldots, 3-\Delta E, 3\}$.

4.2 The Price Process

We test two different stochastic processes for P_t:

Sinusoidal:

$$P_{t+\Delta t} = \mu^P_{t+\Delta t} + \hat{P}_{0,t+\Delta t} \; \forall t \in \mathcal{T} \setminus \{T\},$$

where $\mu^P_t = 40 - 10\sin\left(\frac{5\pi t}{2T}\right)$ and $\hat{P}_{0,t} \sim \mathcal{N}(\mu_P, \sigma_P^2)$.

1st-order Markov chain:

$$P_{t+\Delta t} = P_t + \hat{P}_{0,t+\Delta t} + \mathbb{1}_{\{u_{t+\Delta t} \leq p\}} \hat{P}_{1,t+\Delta t} \; \forall t \in \mathcal{T} \setminus \{T\},$$

such that $P^{min} \leq P_t \leq P^{max}$, and where $\hat{P}_{0,t}$ is either pseudonormally or uniformly distributed as indicated in Table 2. In the case where $\hat{P}_{0,t} \sim \mathcal{N}(\mu_P, \sigma_P^2)$, its support is $\{-8, -8+\Delta P, -8+2\Delta P, \ldots, 0, \ldots, 8-\Delta P, 8\}$. We let $u_t \sim \mathcal{U}(0,1)$, and we let $p = 0.031$ for problems where jumps may occur and $p = 0$ otherwise, and $\hat{P}_{1,t} \sim \mathcal{N}(0, 50^2)$ with support $\{-40, -40+\Delta P, -40+2\Delta P, \ldots, 0, \ldots, 40-\Delta P, 40\}$.

4.3 The Demand Process

The demand is assumed to be deterministic and given by $D_t = \lfloor \max\left[0, 3 - 4\sin\left(\frac{2\pi t}{T}\right)\right] \rfloor$, where $\lfloor \cdot \rfloor$ is the floor function.

4.4 The Markov Decision Process (MDP)

The optimal solution to stochastic problems can be found for problems which have denumerable and relatively small state, decision and outcome spaces, $\mathcal{S}_t$, $\mathcal{X}_t$ and $\mathcal{W}_t$, respectively. In these cases, Bellman's optimality equation can be written as:

$$V_t^*(S_t) = \max_{x_t \in \mathcal{X}_t} \Big(C(S_t, x_t) + \sum_{s'=1}^{|\mathcal{S}_t|} \mathbb{P}_t(s'|S_t, x_t) V^*_{t+\Delta t}(s') \Big) \text{ for } t \in \mathcal{T}, \tag{8}$$

where $\mathbb{P}_t(s'|S_t, x_t)$ is the time-dependent conditional transition probability of going from state S_t to state s' given the decision x_t, and where we assume that $V^*_{T+\Delta t} = 0$. After solving (8), we can simulate the model as a MDP by stepping forward in time following the optimal policy, π^*, defined by the optimal value functions $(V_t^*)_{t \in \mathcal{T}}$.

For a given sample path $\omega \in \Omega$, we can simulate the MDP by solving:

$$X_t^{\pi^*}(S_t(\omega)) = \arg\max_{x_t \in \mathcal{X}_t} \Big(C(S_t(\omega), x_t) + \sum_{s'=1}^{|\mathcal{S}_t(\omega)|} \mathbb{P}_t(s'|S_t(\omega), x_t) V^*_{t+\Delta t}(s'|S_t(\omega), x_t) \Big) \text{ for } t \in \mathcal{T},$$

where $S_{t+1}(\omega) = S^M(S_t(\omega), X_t^{\pi^*}(S_t(\omega)), W_{t+1}(\omega))$.

Label	Price, P_t	Wind Energy, E_t	Demand, D_t	$\mathbb{F}^{1000}$
D1	Sinusoidal	Constant	Sinusoidal	99.99%
D2	Sinusoidal	Step	Step	99.92%
D3	Sinusoidal	Step	Sinusoidal	99.97%
D4	Sinusoidal	Sinusoidal	Step	99.98%
D5	Constant	Constant	Sinusoidal	99.97%
D6	Constant	Step	Step	99.93%
D7	Constant	Step	Sinusoidal	99.98%
D8	Constant	Sinusoidal	Step	99.99%
D9	Fluctuating	Fluctuating	Sinusoidal	99.97%
D10	Fluctuating	Fluctuating	Constant	99.96%

Table 1: Deterministic test problems.

	Resource, R_t		Wind, E_t			Price, P_t		
Label	Levels	ΔR	Levels	ΔE	$\hat{E}_t$	Levels	Process	$\hat{P}_{0,t}$
S1	61	0.50	13	0.50	$\mathcal{U}(-1,1)$	7	Sinusoidal	$\mathcal{N}(0,25^2)$
S2	61	0.50	13	0.50	$\mathcal{N}(0,0.5^2)$	7	Sinusoidal	$\mathcal{N}(0,25^2)$
S3	61	0.50	13	0.50	$\mathcal{N}(0,1.0^2)$	7	Sinusoidal	$\mathcal{N}(0,25^2)$
S4	61	0.50	13	0.50	$\mathcal{N}(0,1.5^2)$	7	Sinusoidal	$\mathcal{N}(0,25^2)$
S5	31	1.00	7	1.00	$\mathcal{U}(-1,1)$	41	1st-order + jump	$\mathcal{N}(0,0.5^2)$
S6	31	1.00	7	1.00	$\mathcal{U}(-1,1)$	41	1st-order + jump	$\mathcal{N}(0,1.0^2)$
S7	31	1.00	7	1.00	$\mathcal{U}(-1,1)$	41	1st-order + jump	$\mathcal{N}(0,2.5^2)$
S8	31	1.00	7	1.00	$\mathcal{U}(-1,1)$	41	1st-order + jump	$\mathcal{N}(0,5.0^2)$
S9	31	1.00	7	1.00	$\mathcal{N}(0,0.5^2)$	41	1st-order + jump	$\mathcal{N}(0,5.0^2)$
S10	31	1.00	7	1.00	$\mathcal{N}(0,1.0^2)$	41	1st-order + jump	$\mathcal{N}(0,5.0^2)$
S11	31	1.00	7	1.00	$\mathcal{N}(0,1.5^2)$	41	1st-order + jump	$\mathcal{N}(0,5.0^2)$
S12	31	1.00	7	1.00	$\mathcal{N}(0,2.0^2)$	41	1st-order + jump	$\mathcal{N}(0,5.0^2)$
S13	31	1.00	7	1.00	$\mathcal{N}(0,0.5^2)$	41	1st-order + jump	$\mathcal{N}(0,1.0^2)$
S14	31	1.00	7	1.00	$\mathcal{N}(0,1.0^2)$	41	1st-order + jump	$\mathcal{N}(0,1.0^2)$
S15	31	1.00	7	1.00	$\mathcal{N}(0,1.5^2)$	41	1st-order + jump	$\mathcal{N}(0,1.0^2)$
S16	31	1.00	7	1.00	$\mathcal{N}(0,0.5^2)$	41	1st-order	$\mathcal{N}(0,1.0^2)$
S17	31	1.00	7	1.00	$\mathcal{N}(0,1.0^2)$	41	1st-order	$\mathcal{N}(0,1.0^2)$
S18	31	1.00	7	1.00	$\mathcal{N}(0,1.5^2)$	41	1st-order	$\mathcal{N}(0,1.0^2)$
S19	31	1.00	7	1.00	$\mathcal{N}(0,0.5^2)$	41	1st-order	$\mathcal{N}(0,5.0^2)$
S20	31	1.00	7	1.00	$\mathcal{N}(0,1.0^2)$	41	1st-order	$\mathcal{N}(0,5.0^2)$
S21	31	1.00	7	1.00	$\mathcal{N}(0,1.5^2)$	41	1st-order	$\mathcal{N}(0,5.0^2)$

Table 2: Stochastic test problems.

Since the transition from S_t to s' is stochastic, a statistical estimate of (7) may be found by simulating K sample paths $\omega^1, \dots, \omega^K \in \Omega$:

$$\overline{F} = \frac{1}{K} \sum_{k=1}^{K} \sum_{t \in \mathcal{T}} C\left(S_t(\omega^k), X_t^{\pi^*}(S_t(\omega^k))\right).$$

5 The Benchmarks

The set of deterministic benchmark problems in given in table 1. That of stochastic benchmark problems is given in Table 2. Each problem is given a *label*. Some parameters are fixed for all problem instances where they are used. These are given in tables 3 and 4.

5.1 The MATLAB Files

Deterministic problems:

The data for each problem instance is in a structure named *label*. Each structure constains six components:

Parameter	Value
R^{max}	100.00
R^{min}	0.00
R_0	0.00
η^c	0.90
η^d	0.90
γ^c	0.10
γ^d	0.10
T	2000
Δt	1

Table 3: List of parameters for deterministic test problems.

Parameter	Value
R^{max}	30.00
R^{min}	0.00
R_0	25.00
P^{max}	70.00
P^{min}	30.00
ΔP	1.00
E^{max}	7.00
E^{min}	1.00
η^c	1.00
η^d	1.00
γ^c	5.00
γ^d	5.00
T	100
Δt	1
K	256

Table 4: List of parameters for stochastic test problems.

label.C A scalar C, where $C = F^*$ represents the optimal solution.

label.R A $(T+1) \times 1$ vector R, where R_t represents the energy in storage at time t.

label.e A $(T+1) \times 1$ vector E, where E_t represents the energy available from wind at time t.

label.p A $(T+1) \times 1$ vector P, where P_t represents the electricity price at time t.

label.D A $(T+1) \times 1$ vector D, where D_t represents the energy demand at time t.

label.x A $6 \times (T+1)$ matrix $\mathbf{x}$, where $\mathbf{x}_{:t}$ represents the optimal decision vector at time t.

Stochastic problems:

The data for each problem instance is in a structure named *label*, contained in *label.mat*. Each structure constains eight components:

label.C A $(T+1) \times K$ matrix $\mathbf{C}$, where $\mathbf{C}_{tk}$ represents the contribution earned at time t for sample path k, i.e. $\mathbf{C}_{tk} = C\left(S_t(\omega^k), X_t^{\pi^*}(S_t(\omega^k))\right)$.

label.R A $(T+1) \times K$ matrix $\mathbf{R}$, where $\mathbf{R}_{tk}$ represents the energy in storage at time t for sample path k.

label.e A $(T+1) \times K$ matrix $\mathbf{E}$, where $\mathbf{E}_{tk}$ represents the energy available from wind at time t for sample path k.

label.ehat A $(T+1) \times K$ matrix $\hat{\mathbf{E}}$, where $\hat{\mathbf{E}}_{tk}$ represents the change in energy available from wind between times $t-1$ and t for sample path k.

label.p A $(T+1) \times K$ matrix $\mathbf{P}$, where $\mathbf{P}_{ij}$ represents the electricity price at time i for sample path j.

label.phat A $(T+1) \times K$ matrix $\hat{\mathbf{P}}$, where $\hat{\mathbf{P}}_{tk}$ represents the change in electricity price between times $t-1$ and t for sample path k.

label.D A $(T+1) \times 1$ vector D, where D_t represents the energy demand at time t.

label.x A $6 \times (T+1) \times K$ tensor $\mathbf{x}$, where $\mathbf{x}_{:tk}$ represents the optimal decision vector at time t for sample path k, $X_t^{\pi^*}(S_t(\omega_k))$.

5.2 The *.txt* Files

Deterministic problems:

The data for each problem instance is in a folder named *label*. In the folder, there are six *.txt* files, one for each of the following components:

C.*txt* A scalar C, where $C = F^*$ represents the optimal solution.

R.*txt* A $(T+1) \times 1$ vector R, where R_t represents the energy in storage at time t.

e.*txt* A $(T+1) \times 1$ vector E, where E_t represents the energy available from wind at time t.

p.*txt* A $(T+1) \times 1$ vector P, where P_t represents the electricity price at time t.

D.*txt* A $(T+1) \times 1$ vector D, where D_t represents the energy demand at time t.

x.*txt* A $6 \times (T+1)$ matrix $\mathbf{x}$, where $\mathbf{x}_{:t}$ (the t*th* column) represents the optimal decision vector at time t.

Stochastic problems:

The data for each problem instance is in a folder named *label*. In the folder, there are 263 *.txt* files, one for each of the following components:

C.*txt* A $(T+1) \times K$ matrix $\mathbf{C}$, where $\mathbf{C}_{tk}$ represents the contribution earned at time t for sample path k, i.e. $\mathbf{C}_{tk} = C\left(S_t(\omega^k), X_t^{\pi^*}(S_t(\omega^k))\right)$.

R.*txt* A $(T+1) \times K$ matrix $\mathbf{R}$, where $\mathbf{R}_{tk}$ represents the energy in storage at time t for sample path k.

e.*txt* A $(T+1) \times K$ matrix $\mathbf{E}$, where $\mathbf{E}_{tk}$ represents the energy available from wind at time t for sample path k.

ehat.*txt* A $(T+1) \times K$ matrix $\hat{\mathbf{E}}$, where $\hat{\mathbf{E}}_{tk}$ represents the change in energy available from wind between times $t-1$ and t for sample path k.

p.*txt* A $(T+1) \times K$ matrix $\mathbf{P}$, where $\mathbf{P}_{ij}$ represents the electricity price at time i for sample path j.

phat.*txt* A $(T+1) \times K$ matrix $\hat{\mathbf{P}}$, where $\hat{\mathbf{P}}_{tk}$ represents the change in electricity price between times $t-1$ and t for sample path k.

D.*txt* A $(T+1) \times 1$ vector D, where D_t represents the energy demand at time t.

x*k*.*txt* A $6 \times (T+1)$ matrix $\mathbf{x}^k$, where $\mathbf{x}^k{}_{:t}$ represents the optimal decision vector at time t for sample path k, $X_t^{\pi^*}(S_t(\omega_k))$.